

AP Calculus AB - Continuity and the Intermediate Value Theorem - Answer Key

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This answer key gives a full worked solution for every problem in the continuity worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 1.10, AP CALC AB 1.11, AP CALC AB 1.12, AP CALC AB 1.16**

Exercise 1 - Continuity of a polynomial

(1) $f(1)=1+3=4$ is defined. (2) $\lim_{x \rightarrow 1}(x^2+3x)=4$ exists by substitution. (3) The limit equals $f(1)$.

Final answer: **yes, f is continuous at $x = 1$.**

Exercise 2 - Where is it undefined?

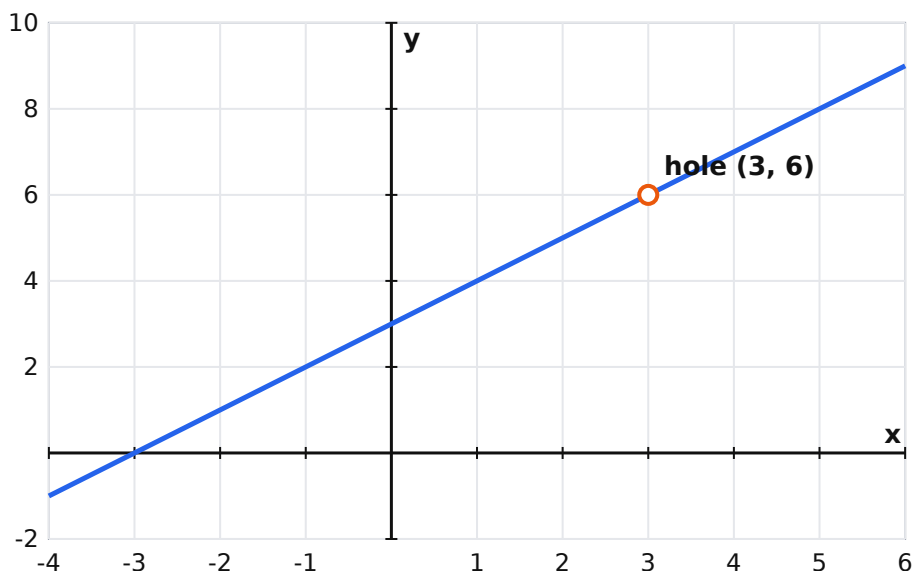
The denominator is zero at $x=4$, where the numerator is $5 \neq 0$. The outputs grow without bound.

Final answer: **an infinite discontinuity at $x = 4$** (a vertical asymptote).

Exercise 3 - Removable discontinuity

Factor: $\frac{(x-3)(x+3)}{x-3} = x+3$ for $x \neq 3$, so $\lim_{x \rightarrow 3} h(x) = 6$ while $h(3)$ is undefined.

Final answer: **removable**; define $h(3)=6$.



The graph has a hole at (3, 6).

Exercise 4 - Types from a graph

At $x=1$: the left-hand limit is 1.4 and the right-hand limit is 2.5. The one-sided limits differ, so it is a **jump discontinuity**.

At $x=3$: both sides approach 1.5 but the point is at height 3. This is a **removable discontinuity**.

Exercise 5 - Checking conditions from data

Condition 1 holds: $f(4)=7$. Condition 2 fails: the one-sided limits 7 and 9 differ, so the limit does not exist.

Final answer: **not continuous**; condition 2 fails (a jump).

Exercise 6 - Repairing a hole

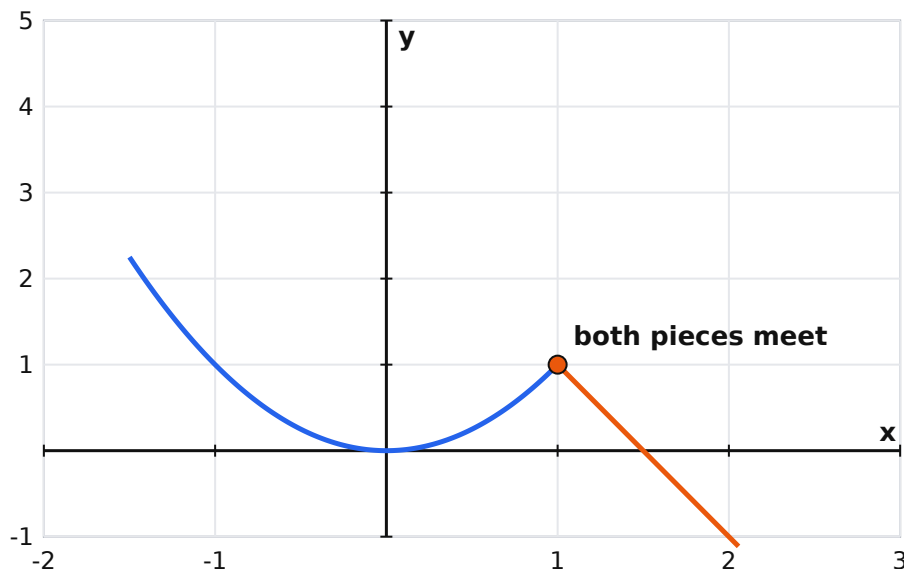
Factor: $\frac{(x+3)(x-2)}{x-2} = x+3$, so $\lim_{x \rightarrow 2} f(x) = 5$.

Continuity requires $f(2)=5$. Final answer: $k=5$.

Exercise 7 - Piecewise, one unknown

Left-hand limit: $1^2=1$. Right-hand value: $a+3$.

Set equal: $a+3=1$, so $a=-2$.



With $a = -2$ the pieces meet at $(1, 1)$.

Exercise 8 - Piecewise, a second example

Left: $2(3)+b=6+b$. Right: $3^2-1=8$.

Set equal: $6+b=8$, so $b=2$.

Exercise 9 - Two points to check

Factor: $p(x) = \frac{x(x-4)}{(x-4)(x+4)}$.

At $x=4$ the factor $x-4$ cancels, leaving $\frac{x}{x+4} \rightarrow \frac{4}{8} = \frac{1}{2}$: **removable**.

At $x=-4$ the factor $x+4$ stays in the denominator with numerator $-4 \cdot (-8) = 32 \neq 0$: **infinite**.

Exercise 10 - Domain and continuity

The function is defined for $x \geq 2$ and is continuous at every interior point. At $x=2$, the left side is not in the domain, so we use the one-sided limit: $\lim_{x \rightarrow 2^+} \sqrt{x-2} = 0 = f(2)$.

Final answer: continuous on $[2, \infty)$ (right-continuous at $x=2$).

Exercise 11 - Sine over x

Since $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, continuity needs $g(0) = 1$.

If $g(0) = 0$, the limit 1 does not equal $g(0)$, so g has a removable discontinuity.

Final answer: $g(0) = 1$.

Exercise 12 - IVT and a cubic

f is a polynomial, so it is continuous on $[1, 2]$.

$f(1) = 1 + 1 - 5 = -3$ and $f(2) = 8 + 2 - 5 = 5$. Since 0 is between -3 and 5 , the IVT gives a c in $(1, 2)$ with $f(c) = 0$.

Final answer: **a zero exists in $(1, 2)$.**

Exercise 13 - Two zeros

The polynomial is continuous everywhere. $f(0) = 1$, $f(1) = 1 - 4 + 1 = -2$ and $f(3) = 27 - 12 + 1 = 16$.

Since $f(0) > 0 > f(1)$, the IVT gives a zero in $(0, 1)$. Since $f(1) < 0 < f(3)$, it gives another zero in $(1, 3)$.

Final answer: **two zeros, one in each interval.**

Exercise 14 - Which values are guaranteed?

4 is between -2 and 10 , so yes, some c in $[1, 5]$ has $f(c) = 4$.

12 is outside $[-2, 10]$, so the IVT does not guarantee it (the function might still reach 12, but the theorem does not say so).

Final answer: $f(c) = 4$ is guaranteed; $f(c) = 12$ is not.

Exercise 15 - When the IVT fails

The IVT requires f to be continuous on all of $[-1, 1]$. But f is undefined at $x = 0$, which lies in the interval, and has an infinite discontinuity there.

Final answer: **the hypothesis of continuity on $[-1, 1]$ is not met**, so the theorem does not apply.

Exercise 16 - Several discontinuities

Factor: $\frac{(x-1)(x+1)}{(x-2)(x-3)}$. No factor cancels.

At $x=2$: numerator $3 \neq 0$. At $x=3$: numerator $8 \neq 0$.

Final answer: **infinite discontinuities at $x = 2$ and $x = 3$** .

Exercise 17 - Shipping charges

At $w=1$: left limit 4, right limit 6. At $w=2$: left limit 6, right limit 8.

Final answer: **jump discontinuities at $w = 1$ and $w = 2$** , each of size $\$2$.

Exercise 18 - Hiking elevation

Elevation is continuous, and 1500 lies between 800 and 2300. By the IVT, she was at 1,500 feet at least once.

The value 2500 is above both endpoint values, so the IVT does not guarantee it; she may never have gone that high.

Final answer: **yes for 1,500 ft; not guaranteed for 2,500 ft**.

Exercise 19 - Two unknowns

At $x=1$: $a+b=2$. At $x=3$: $3a+b=6$.

Subtract: $2a=4$, so $a=2$, and then $b=0$.

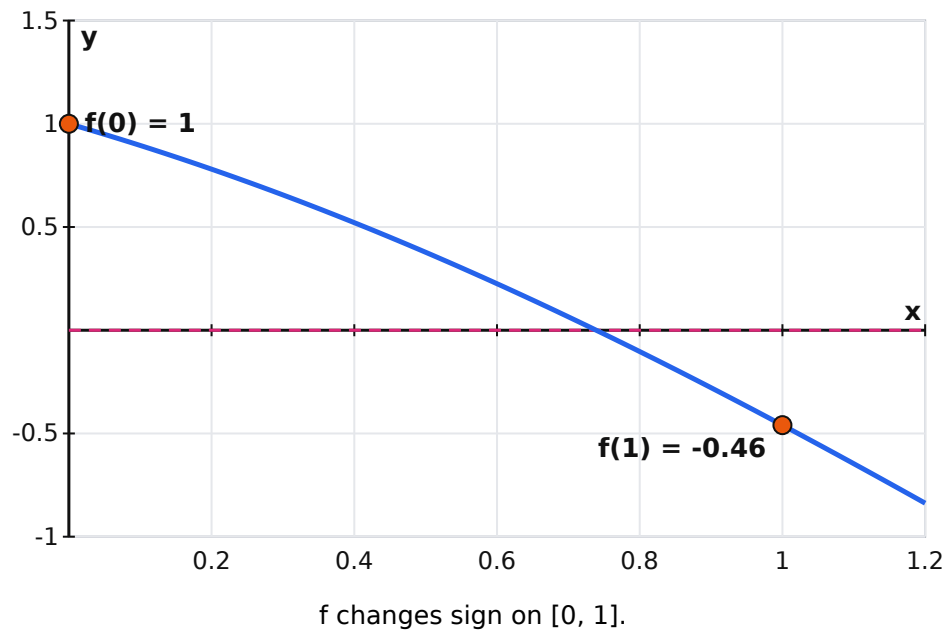
Final answer: $a=2$, $b=0$.

Exercise 20 - Solving $\cos x = x$

Let $f(x) = \cos x - x$, which is continuous. $f(0) = 1 - 0 = 1 > 0$ and $f(1) = \cos 1 - 1 \approx -0.4597 < 0$.

By the IVT there is a c in $(0,1)$ with $f(c)=0$, that is, $\cos c = c$.

Final answer: **a solution exists in $(0, 1)$.**



Exercise 21 - Solving an exponential equation

Let $g(x) = e^x + x - 3$, continuous for all x . $g(0) = 1 + 0 - 3 = -2$ and $g(1) = e + 1 - 3 = e - 2 \approx 0.718$.

Since $g(0) < 0 < g(1)$, the IVT gives c in $(0,1)$ with $g(c) = 0$, so $e^c + c = 3$.

Final answer: **a solution exists in $(0, 1)$.**