

AP Calculus AB - Continuity and the Intermediate Value Theorem - Worksheet

Common Core AP CALC AB 1.10, AP CALC AB 1.11, AP CALC AB 1.12, AP CALC AB 1.16 · maths-pdf.fr/en/

Name: _____ Class: _____ Date: ____/____/____

These 21 problems practice continuity, types of discontinuity, making piecewise functions continuous and the Intermediate Value Theorem (AP Calculus AB, topics 1.10, 1.11, 1.12, 1.16). The early problems build the basic skill, and each one uses a different function or situation so that no two feel alike.

Later problems use piecewise functions with unknown constants, proving that equations have solutions, and situations such as shipping charges and a hiking trail. Work in pencil, show every step, and check the answer key only after a full attempt.

AP Calculus AB · Common Core State Standards: **AP CALC AB 1.10, AP CALC AB 1.11, AP CALC AB 1.12, AP CALC AB 1.16**

Exercise 1 - Continuity of a polynomial

Is $f(x) = x^2 + 3x$ continuous at $x = 1$? Check the three conditions.

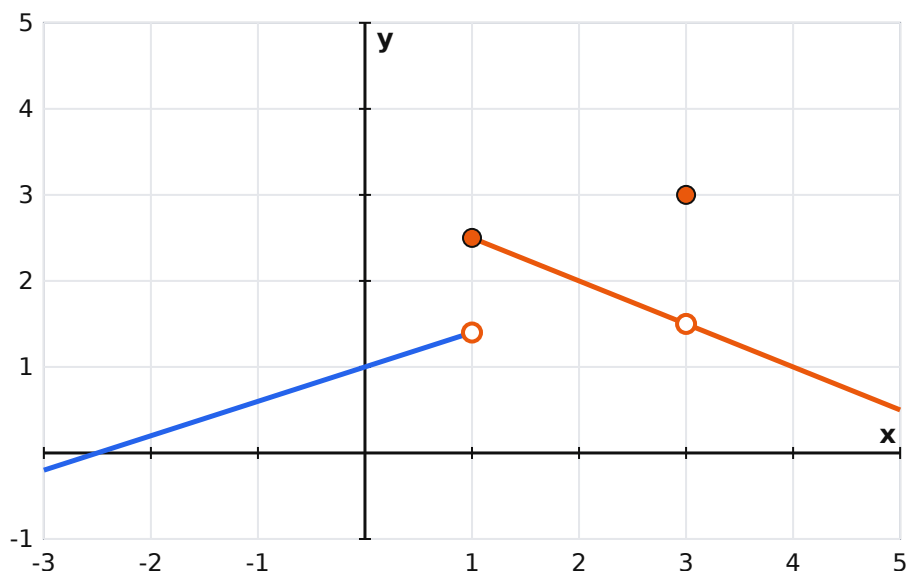
Exercise 2 - Where is it undefined?

Find every x -value where $g(x) = \frac{x+1}{x-4}$ is discontinuous, and classify the discontinuity.

Exercise 3 - Removable discontinuity

Classify the discontinuity of $h(x) = \frac{x^2-9}{x-3}$ at $x = 3$ and say how to define $h(3)$ to make h continuous.

Exercise 4 - Types from a graph



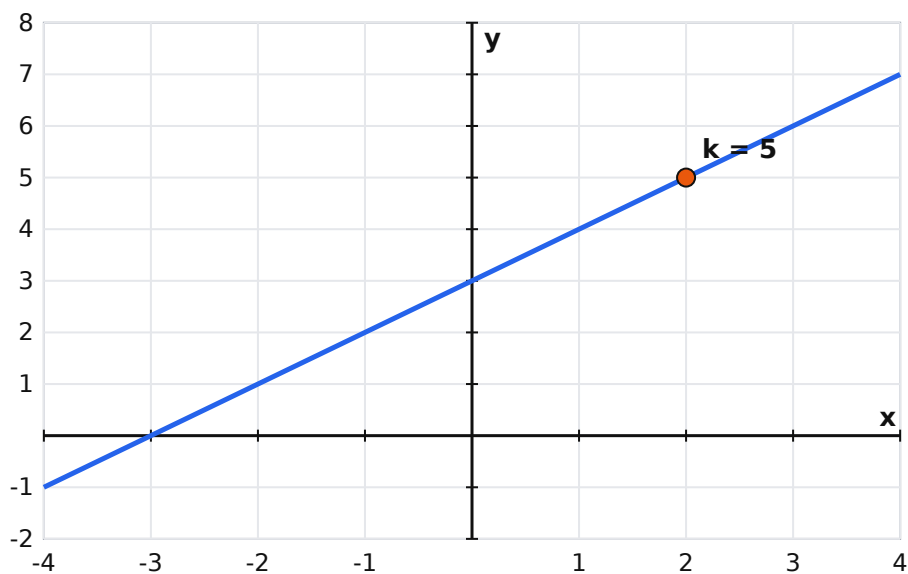
The graph of f has trouble at $x = 1$ and $x = 3$.

Identify each discontinuity of f in the graph and classify it as removable, jump or infinite.

Exercise 5 - Checking conditions from data

A function has $f(4) = 7$, $\lim_{x \rightarrow 4^-} f(x) = 7$ and $\lim_{x \rightarrow 4^+} f(x) = 9$. Is f continuous at $x = 4$? Which condition fails?

Exercise 6 - Repairing a hole



The repaired graph passes through $(2, 5)$.

Let $f(x) = \frac{x^2 + x - 6}{x - 2}$ for $x \neq 2$ and $f(2) = k$. Find k so that f is continuous at $x = 2$.

Exercise 7 - Piecewise, one unknown

Let $f(x) = x^2$ for $x < 1$ and $f(x) = ax + 3$ for $x \geq 1$. Find a so that f is continuous at $x = 1$.

Exercise 8 - Piecewise, a second example

Let $f(x) = 2x + b$ for $x \leq 3$ and $f(x) = x^2 - 1$ for $x > 3$. Find b for continuity at $x = 3$.

Exercise 9 - Two points to check

For $p(x) = \frac{x^2 - 4x}{x^2 - 16}$, classify the discontinuities at $x = 4$ and $x = -4$.

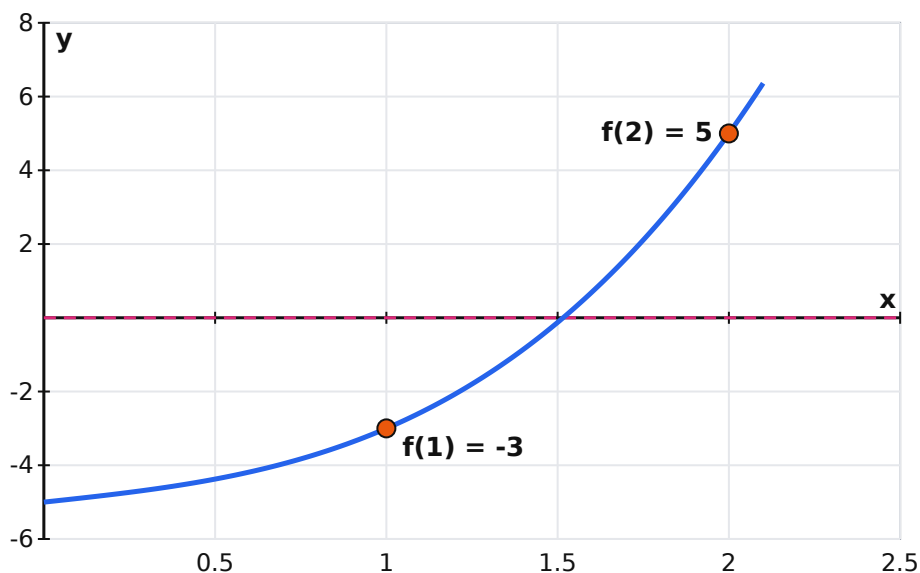
Exercise 10 - Domain and continuity

On what interval is $f(x) = \sqrt{x-2}$ continuous? Is it continuous at $x = 2$?

Exercise 11 - Sine over x

Define $g(x) = \frac{\sin x}{x}$ for $x \neq 0$. What value of $g(0)$ makes g continuous at 0? What if $g(0) = 0$?

Exercise 12 - IVT and a cubic



$f(1)$ is negative and $f(2)$ is positive.

Show that $f(x) = x^3 + x - 5$ has a zero in $(1, 2)$.

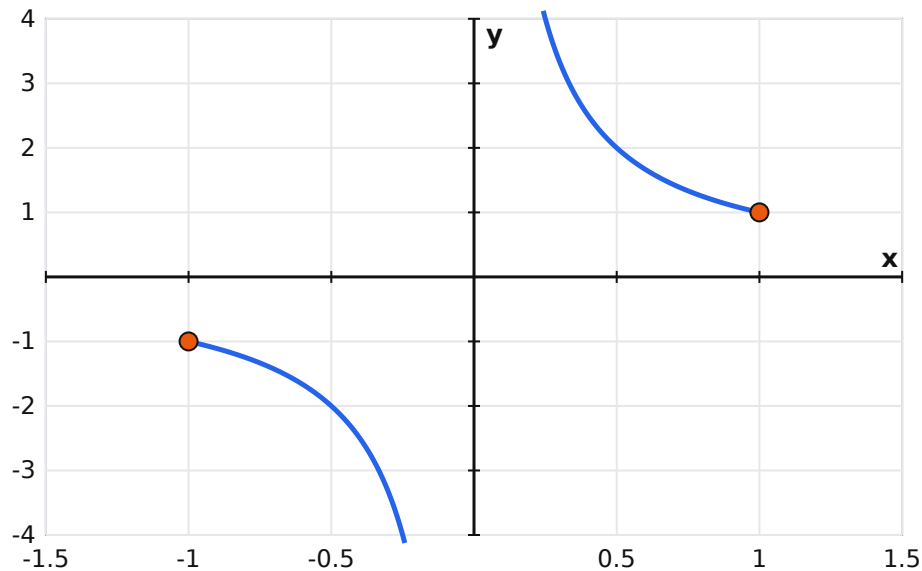
Exercise 13 - Two zeros

Show that $f(x) = x^3 - 4x + 1$ has a zero in $(0, 1)$ and another in $(1, 3)$.

Exercise 14 - Which values are guaranteed?

A function f is continuous on $[1, 5]$ with $f(1) = -2$ and $f(5) = 10$. Does the IVT guarantee a c with $f(c) = 4$? With $f(c) = 12$?

Exercise 15 - When the IVT fails



The graph of $1/x$ jumps across $x = 0$.

For $f(x) = \frac{1}{x}$, $f(-1) = -1$ and $f(1) = 1$, yet $f(x) = 0$ has no solution. Why does this not contradict the IVT?

Exercise 16 - Several discontinuities

Find and classify every discontinuity of $r(x) = \frac{x^2 - 1}{x^2 - 5x + 6}$.

Exercise 17 - Shipping charges

Sofia's shop charges \$4 to ship a package up to 1 pound, \$6 for over 1 up to 2 pounds, and \$8 for over 2 up to 3 pounds. Where is the shipping cost $S(w)$ discontinuous for $0 < w \leq 3$? Classify and give the size of each jump.

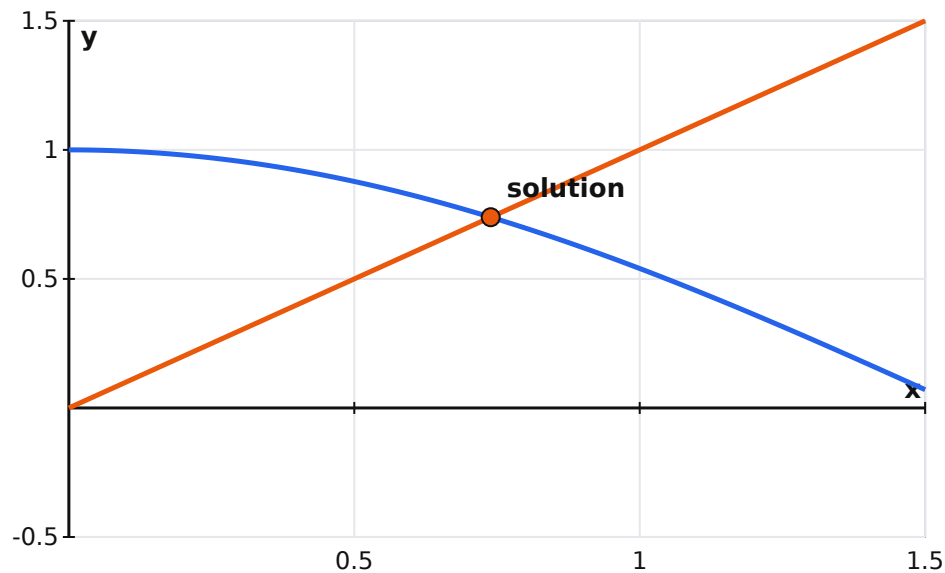
Exercise 18 - Hiking elevation

Mei hikes a trail where her elevation is a continuous function of time. She starts at 800 feet and finishes at 2,300 feet. Must she have been at exactly 1,500 feet at some time? Exactly 2,500 feet?

Exercise 19 - Two unknowns

Let $f(x)=2$ for $x \leq 1$, $f(x)=ax+b$ for $1 < x < 3$ and $f(x)=6$ for $x \geq 3$. Find a and b so that f is continuous everywhere.

Exercise 20 - Solving $\cos x = x$



The curves $y = \cos x$ and $y = x$ cross once on $[0, 1]$.

Show that the equation $\cos x = x$ has a solution in $[0, 1]$ (radians).

Exercise 21 - Solving an exponential equation

Show that $e^x + x = 3$ has a solution in $(0, 1)$.