

AP Calculus AB - The Derivative as a Limit - Lesson

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The derivative of f at $x = a$ is the limit of the difference quotient $(f(a + h) - f(a))/h$ as h approaches 0. It gives the slope of the tangent line and the instantaneous rate of change. A function is not differentiable at corners, cusps, vertical tangents, or discontinuities, but every differentiable function is continuous.

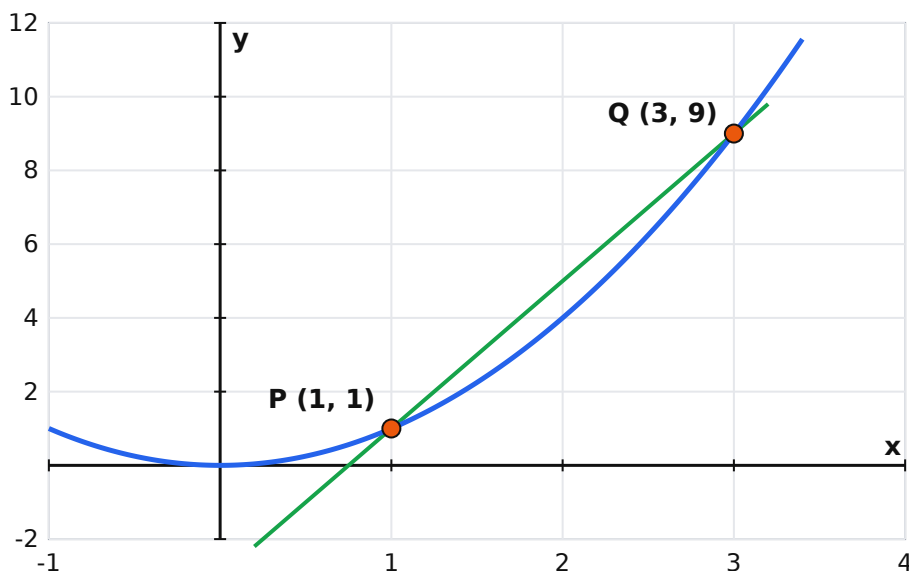
AP Calculus AB · Common Core State Standards: **AP CALC AB 2.1, AP CALC AB 2.2, AP CALC AB 2.3, AP CALC AB 2.4**

Key facts

- The derivative is the limit of secant slopes, which are average rates of change.
- $f'(a)$ is the slope of the tangent line at $x = a$; the tangent is $y - f(a) = f'(a)(x - a)$.
- Both forms of the definition give the same value when the limit exists.
- Differentiable implies continuous, but continuous does not imply differentiable.

What is the derivative?

The **derivative** measures how fast a function changes at a single instant. It is the slope of the tangent line to the graph. Calculus builds it from average rates of change, which are slopes of secant lines.



The secant line through P and Q has slope $(9 - 1)/(3 - 1) = 4$.

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Limit definition of the derivative

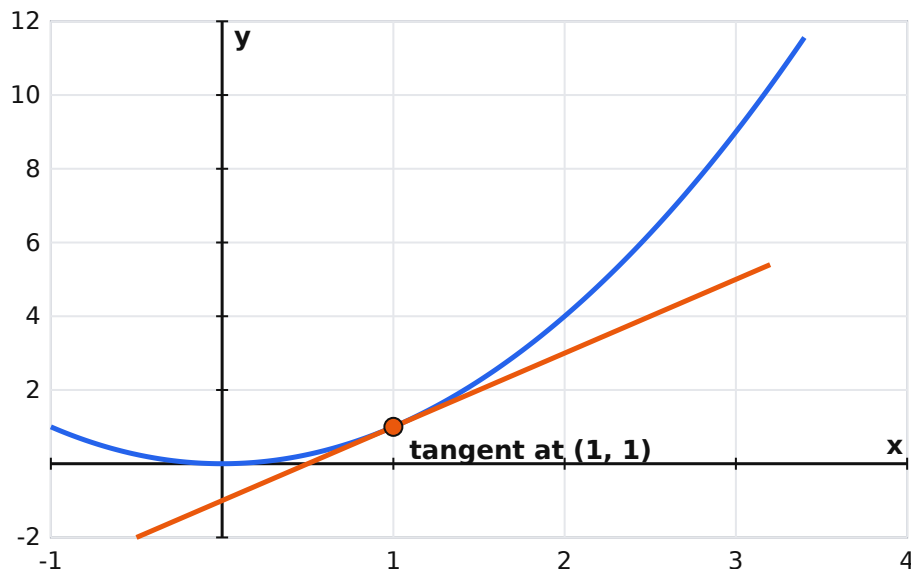
$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \text{ or, equivalently, } f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}, \text{ provided the limit exists.}$$

How do secant slopes become a tangent slope?

Take a second point at distance h from the first. The average rate of change over $[a, a+h]$ is the secant slope. Letting $h \rightarrow 0$ slides the second point toward the first.

h	1	0.1	0.01	0.001
slope	3	2.1	2.01	2.001

The secant slopes approach 2 as h shrinks.



As Q slides toward P , the secant becomes the tangent, with slope $f'(1) = 2$.

How do you compute a derivative from the definition?

1. Write the difference quotient $\frac{f(a+h) - f(a)}{h}$.
2. Expand $f(a+h)$ and simplify the numerator.
3. Cancel the factor h from every term.
4. Take the limit as $h \rightarrow 0$.

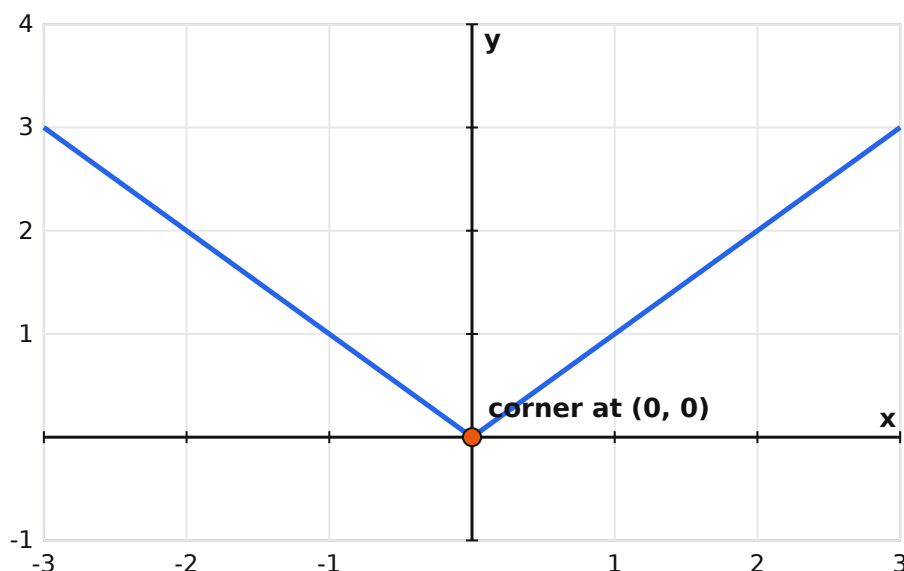
Example. For $f(x) = x^2$ at $a = 1$: $\frac{(1+h)^2 - 1}{h} = \frac{2h + h^2}{h} = 2 + h \rightarrow 2$. So $f'(1) = 2$, and the tangent line is $y - 1 = 2(x - 1)$, or $y = 2x - 1$.

What does a derivative tell you in a real situation?

The derivative carries the units of output per unit of input. If $s(t)$ is the distance in feet after t seconds, then $s'(3) = 24$ means the object is moving at 24 feet per second at the instant $t = 3$. If $C(x)$ is the cost of x items, then $C'(10)$ estimates the cost of making one more item after 10. Always state the units and the input value when you interpret a derivative.

When is a function not differentiable?

A function is differentiable at $x = a$ only if the tangent line exists and is not vertical. It fails at corners, cusps, vertical tangents and discontinuities.



A corner: the left slope is -1 and the right slope is 1 , so $f'(0)$ does not exist.

Differentiability implies continuity

If f is differentiable at a , then f is continuous at a . The converse is false: $|x|$ is continuous at 0 but not differentiable there.

Questions about the derivative as a limit

What is the limit definition of the derivative?

The derivative $f'(a)$ equals the limit as h approaches 0 of $[f(a + h) - f(a)]/h$. It is the slope of the tangent line at $x = a$, found as the limit of secant slopes.

What is the difference between average and instantaneous rate of change?

An average rate of change is the slope of a secant line over an interval. An instantaneous rate of change is the limit of those slopes as the interval shrinks to a point, which is the derivative.

How do you write the equation of a tangent line?

Find $f(a)$ and $f'(a)$, then use point-slope form: $y - f(a) = f'(a)(x - a)$.

Where is a function not differentiable?

A function is not differentiable at a corner, a cusp, a vertical tangent, or any point of discontinuity. At those points the limit of the difference quotient fails to exist.

Does continuity guarantee differentiability?

No. The function $f(x) = |x|$ is continuous at 0 but has a corner there, so it is not differentiable. Differentiability does guarantee continuity.