

AP Calculus AB - The Derivative as a Limit - Worksheet

Common Core AP CALC AB 2.1, AP CALC AB 2.2, AP CALC AB 2.3, AP CALC AB 2.4 · maths-pdf.fr/en/

Name: _____ Class: _____ Date: ____/____/____

These 22 problems practice average and instantaneous rates of change, the limit definition of the derivative, tangent lines and differentiability (AP Calculus AB, topics 2.1, 2.2, 2.3, 2.4). The early problems build the basic skill, and each one uses a different function or situation so that no two feel alike.

Later problems cover corners, vertical tangents, piecewise functions and real situations such as skateboard speed and marginal cost. Work in pencil, show every step, and check the answer key only after a full attempt.

AP Calculus AB · Common Core State Standards: **AP CALC AB 2.1, AP CALC AB 2.2, AP CALC AB 2.3, AP CALC AB 2.4**

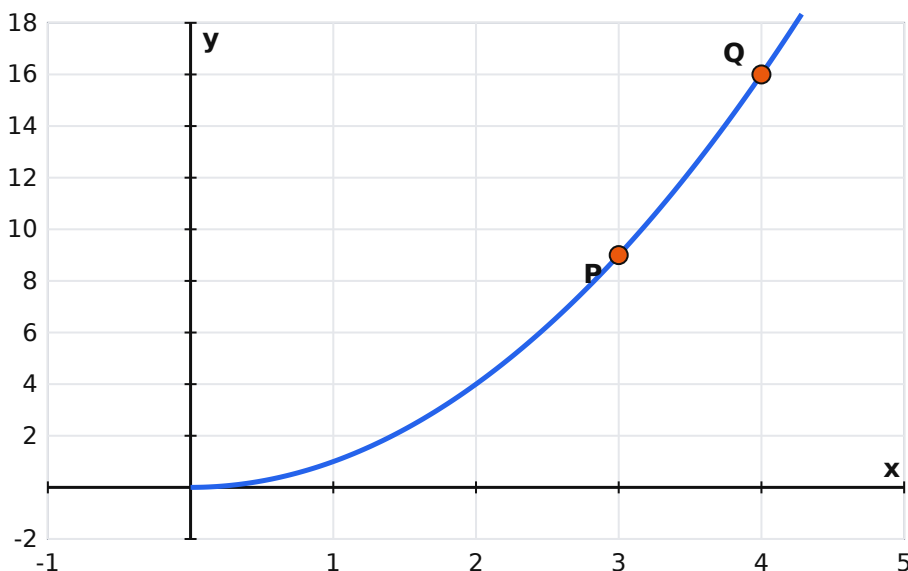
Exercise 1 - Average rate of change

Find the average rate of change of $f(x) = x^2 + 1$ on $[1, 4]$.

Exercise 2 - Secant slope from a table

The table gives $g(2) = 7$, $g(2.5) = 9.5$ and $g(3) = 13$. Find the slope of the secant line over $[2, 2.5]$ and over $[2.5, 3]$. Which interval has the greater average rate?

Exercise 3 - Derivative at a point, x squared



Two points on $y = x^2$: P(3, 9) and Q(4, 16).

Use the limit definition to find $f'(3)$ for $f(x) = x^2$.

Exercise 4 - Derivative at a point, a quadratic

Use the limit definition to find $f'(1)$ for $f(x) = 3x^2 - 2x$.

Exercise 5 - Derivative of a reciprocal

Use the limit definition to find $f'(2)$ for $f(x) = \frac{1}{x}$.

Exercise 6 - Derivative of a square root

Use the limit definition to find $f'(4)$ for $f(x) = \sqrt{x}$.

Exercise 7 - Derivative of a cube

Use the limit definition to find $f'(2)$ for $f(x) = x^3$.

Exercise 8 - The x-approaches-a form

Use $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$ to find $f'(3)$ for $f(x) = x^2 - 5x$.

Exercise 9 - Derivative function and its zeros

Use the limit definition to find $f'(x)$ for $f(x) = 3x^2 - x^3$, and then find where the tangent line is horizontal.

Exercise 10 - Using the derivative function

For $f(x) = 2x^2 - x$, find $f'(x)$ from the definition and then evaluate $f'(-2)$.

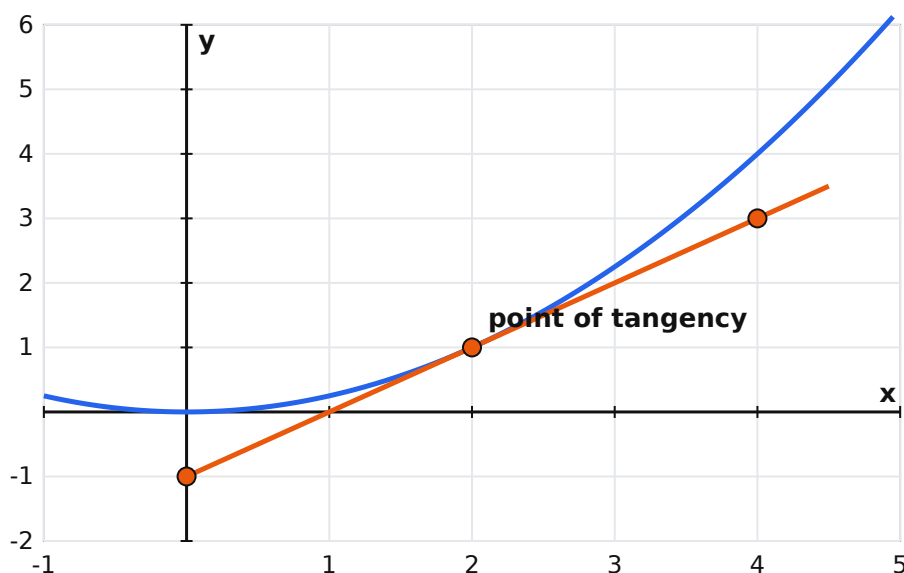
Exercise 11 - Tangent line to a reciprocal

Use the limit definition of the derivative to find the equation of the tangent line to $f(x) = \frac{1}{x}$ at $x = 2$.

Exercise 12 - Tangent line, second example

Find the tangent line to $f(x) = x^2 - 4x + 1$ at $x = 1$.

Exercise 13 - Slope from a graph



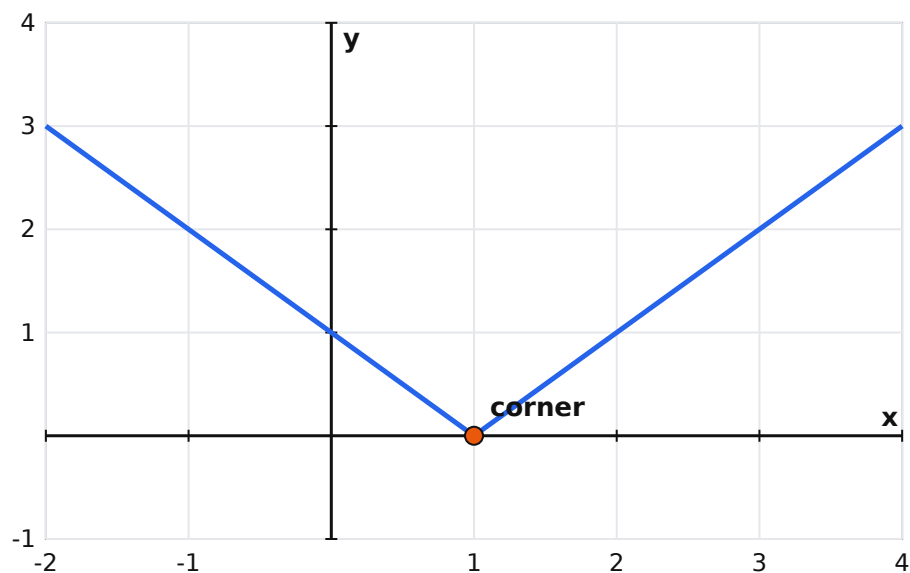
The tangent line at $x = 2$ passes through $(0, -1)$ and $(4, 3)$.

The graph shows a curve and its tangent line at $x = 2$. The tangent passes through $(0, -1)$ and $(4, 3)$. Estimate $f'(2)$ and write the tangent equation.

Exercise 14 - Estimating from a table

A function has $f(1.9) = 4.959$ and $f(2.1) = 7.161$. Estimate $f'(2)$ using the slope of the secant from $x = 1.9$ to $x = 2.1$.

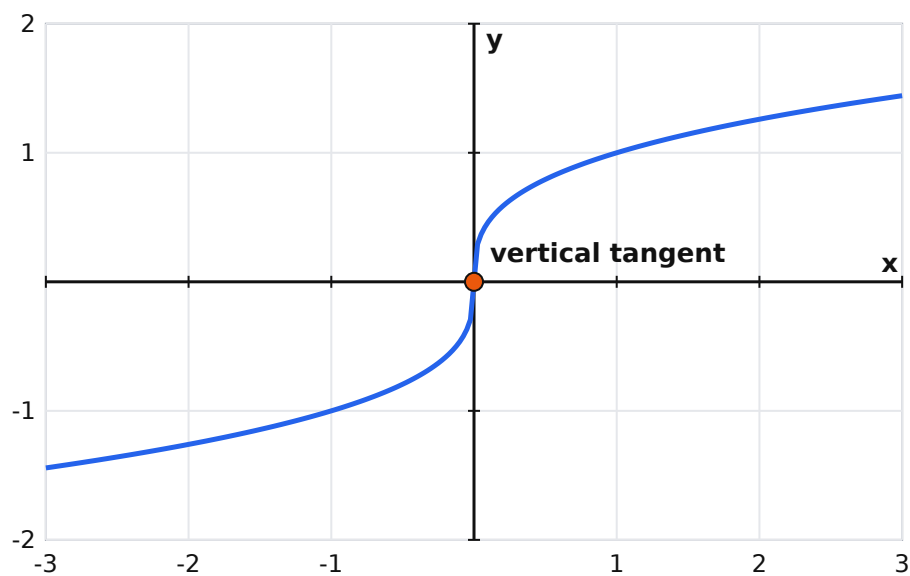
Exercise 15 - A corner



The graph of $f(x) = |x - 1|$.

Explain why $f(x) = |x - 1|$ is not differentiable at $x = 1$ by finding the one-sided difference quotient limits.

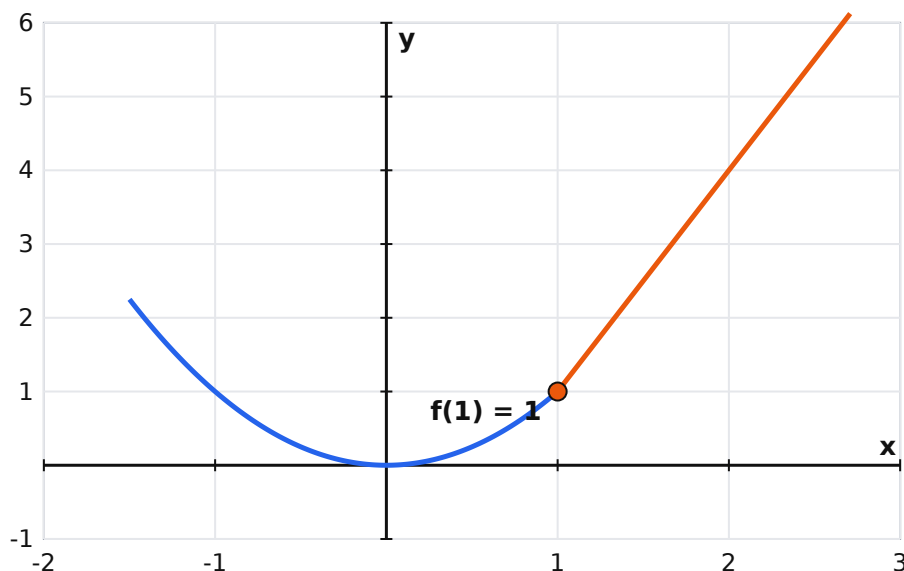
Exercise 16 - A vertical tangent



The graph of $f(x) = \sqrt[3]{x}$.

Show that $f(x) = \sqrt[3]{x}$ is not differentiable at $x = 0$.

Exercise 17 - Continuous but not differentiable



The graph of a piecewise function with a corner at $x = 1$.

Let $f(x) = x^2$ for $x \leq 1$ and $f(x) = 3x - 2$ for $x > 1$. Is f continuous at 1? Is it differentiable at 1?

Exercise 18 - Making a function differentiable

Let $f(x) = x^2$ for $x \leq 2$ and $f(x) = ax + b$ for $x > 2$. Find a and b so that f is differentiable at $x = 2$.

Exercise 19 - Speed from the definition

Diego's skateboard rolls down a ramp so that its distance in feet after t seconds is $s(t) = 4t^2$. Use the limit definition to find the speed at $t = 3$ seconds.

Exercise 20 - Marginal cost

The cost in dollars of making x custom T-shirts is $C(x) = 0.5x^2 + 20x$. Use the limit definition to find $C'(10)$ and interpret it.

Exercise 21 - Absolute value at zero

Use one-sided difference quotients to show that $f(x) = |x|$ is not differentiable at 0, and state $f'(x)$ for $x > 0$ and for $x < 0$.

Exercise 22 - Recognizing a derivative

Each limit is a derivative. Evaluate (a) $\lim_{h \rightarrow 0} \frac{(2+h)^5 - 32}{h}$ and (b) $\lim_{h \rightarrow 0} \frac{\ln(e+h) - 1}{h}$.