

AP Calculus AB - The Derivative as a Limit - Test Answer Key

Common Core AP CALC AB 2.1, AP CALC AB 2.2, AP CALC AB 2.3, AP CALC AB 2.4 · maths-pdf.fr/en/

Use this answer key to score the derivative definition test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

AP Calculus AB · Common Core State Standards: **AP CALC AB 2.1, AP CALC AB 2.2, AP CALC AB 2.3, AP CALC AB 2.4**

Question 1 - Average and instantaneous rate [2 pts]

- a. $\frac{25-4}{3} = 7$.
b. $\frac{4.0401-4}{0.01} = 4.01$, so $f'(2) \approx 4$.

Scoring: 1 point per part.

Question 2 - Limit definition, polynomial [3 pts]

$$f(2) = -2. \quad f(2+h) = 4+4h+h^2-6-3h = -2+h+h^2.$$

$$\frac{h+h^2}{h} = 1+h \rightarrow 1. \text{ Final answer: } f'(2) = 1.$$

Scoring: 1 point for the difference quotient; 1 point for simplifying; 1 point for the limit.

Question 3 - Limit definition, reciprocal [3 pts]

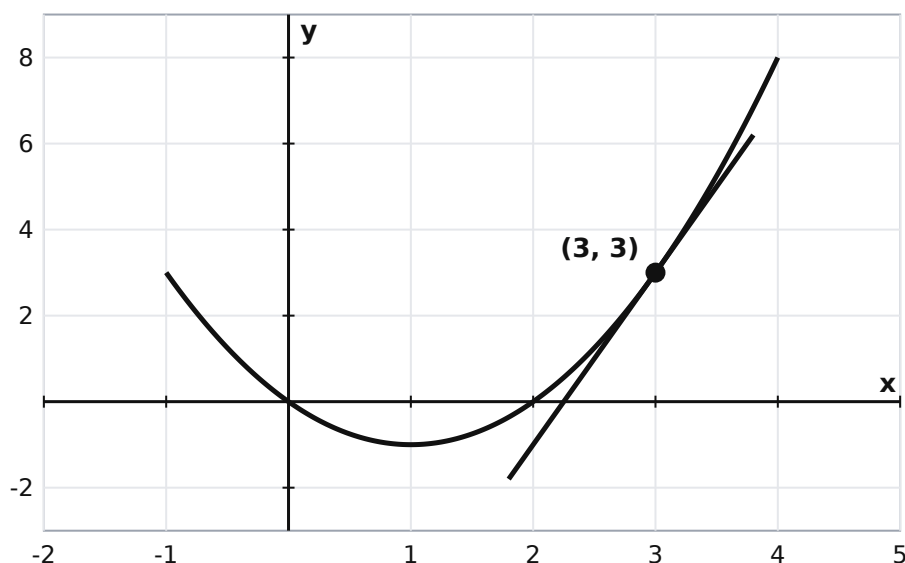
$$\frac{\frac{2}{3+h} - \frac{2}{3}}{h} = \frac{6-2(3+h)}{3(3+h)h} = \frac{-2h}{3(3+h)h} = \frac{-2}{3(3+h)}.$$

$$\text{As } h \rightarrow 0: f'(3) = -\frac{2}{9}.$$

Scoring: 1 point for the difference quotient; 1 point for simplifying; 1 point for the limit.

Question 4 - Tangent line [2 pts]

$g(3) = 9 - 6 = 3$. Tangent: $y - 3 = 4(x - 3)$, so $y = 4x - 9$.

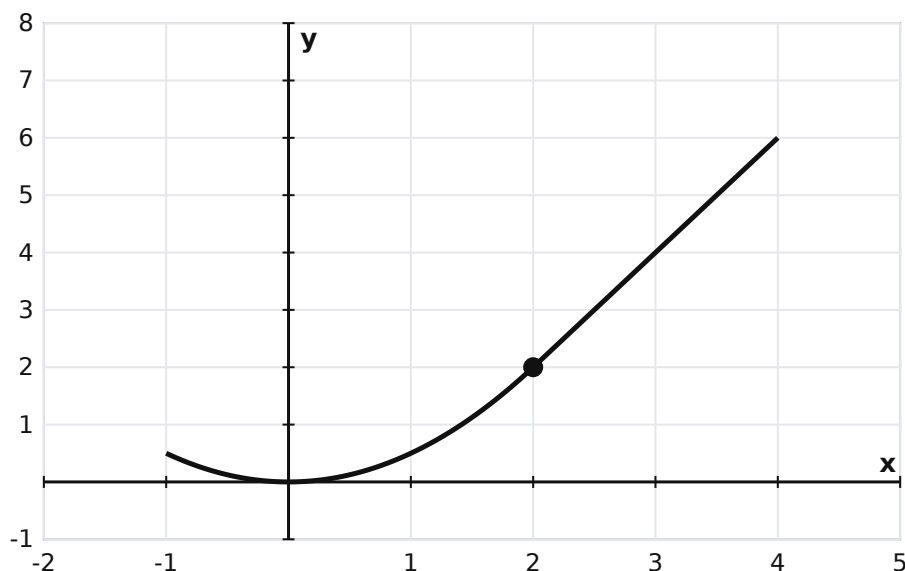


Graph of $g(x) = x^2 - 2x$.

Scoring: 1 point for $g(3)$; 1 point for the equation.

Question 5 - Differentiability of a piecewise function [3 pts]

- a. Left value: $\frac{4}{2} = 2$; right limit: $2(2) - 2 = 2$; $k(2) = 2$. **Continuous.**
b. Left slope: $k'(x) = x$ gives 2. Right slope: 2. The slopes match, so **k is differentiable at $x = 2$** with $k'(2) = 2$.



Scoring: 1 point for (a); 1 point for each one-sided slope in (b) with the conclusion.

Question 6 - Recognizing a derivative [2 pts]

Take $f(x) = x^4$ and $a = 3$, since $f(3) = 81$. Then the limit is $f'(3) = 4 \cdot 27 = 108$.

Final answer: 108.

Scoring: 1 point for identifying f and a ; 1 point for the value.

Question 7 - Cost of a pizza oven [3 pts]

- a. $P(10) = 200$, $P(12) = 216$, so $\frac{216-200}{2} = 8$ dollars per pizza.
- b. $P(10+h) = -(100+20h+h^2)+300+30h = 200+10h-h^2$, so the quotient is $10-h \rightarrow 10$.
 $P'(10) = 10$

Scoring: 1 point for (a); 1 point for the quotient in (b); 1 point for the limit.

Question 8 - Slopes on a graph [2 pts]

At A the curve falls, so the slope is **negative**. At B the curve rises, so **positive**. At C the curve has a peak, so the tangent is horizontal: **zero**.

Scoring: 1 point for A and C correct; 1 point for B correct and a reason for each.

Total

20 points