

AP Calculus AB - Power, Sum and Basic Function Derivative Rules - Lesson

Common Core AP CALC AB 2.5, AP CALC AB 2.6, AP CALC AB 2.7 · maths-pdf.fr/en/

The power rule says the derivative of x^n is $n x^{(n-1)}$, and constants differentiate to 0. Sums, differences and constant multiples are differentiated term by term. The derivatives of $\sin x$, $\cos x$, e^x and $\ln x$ are $\cos x$, $-\sin x$, e^x and $1/x$. A tangent line uses slope $f'(a)$; a normal line uses the negative reciprocal.

AP Calculus AB · Common Core State Standards: **AP CALC AB 2.5, AP CALC AB 2.6, AP CALC AB 2.7**

Key facts

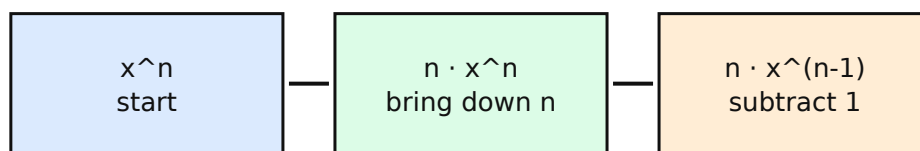
- Power rule: d/dx of x^n equals $n x^{(n-1)}$; the derivative of a constant is 0.
- Rewrite roots and reciprocals as powers before differentiating.
- The derivatives of $\sin x$, $\cos x$, e^x and $\ln x$ are $\cos x$, $-\sin x$, e^x and $1/x$.
- A normal line has slope equal to the negative reciprocal of the tangent slope.

What are the basic derivative rules?

Differentiating from the limit definition is slow. These rules, each proved from the definition, let you differentiate polynomials and roots quickly.

Basic rules

- $\frac{d}{dx}[c] = 0$ for any constant c
- $\frac{d}{dx}[x^n] = n x^{n-1}$ (power rule)
- $\frac{d}{dx}[c f(x)] = c f'(x)$ (constant multiple)
- $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$ (sum and difference)



The power rule in three moves: d/dx of x^n is $n x^{(n-1)}$.

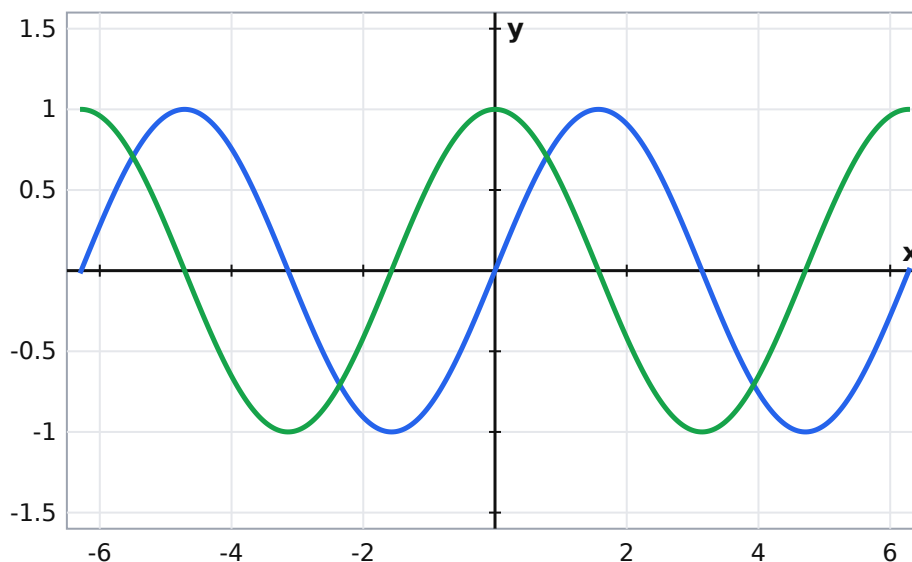
Example. If $f(x) = 4x^3 - 5x^2 + 7x - 9$, then $f'(x) = 12x^2 - 10x + 7$.

How do you differentiate roots and fractions?

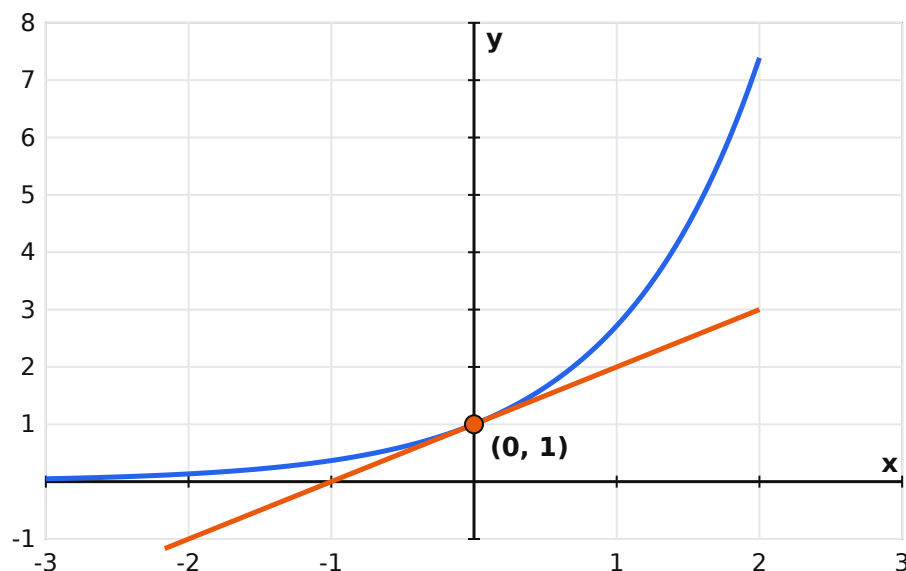
Rewrite each term as a power of x first. Use $\sqrt{x} = x^{1/2}$ and $\frac{1}{x^n} = x^{-n}$. For $f(x) = \sqrt{x} + \frac{5}{x^2}$, write $x^{1/2} + 5x^{-2}$, so $f'(x) = \frac{1}{2}x^{-1/2} - 10x^{-3} = \frac{1}{2\sqrt{x}} - \frac{10}{x^3}$.

What are the derivatives of sine, cosine, e^x and $\ln x$?

Function	Derivative
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
e^x	e^x
$\ln x$	$\frac{1}{x}$



The derivative of $\sin x$ is $\cos x$: the green curve gives the slope of the blue curve.



Since $(e^x)' = e^x$, the slope at $x = 0$ is 1.

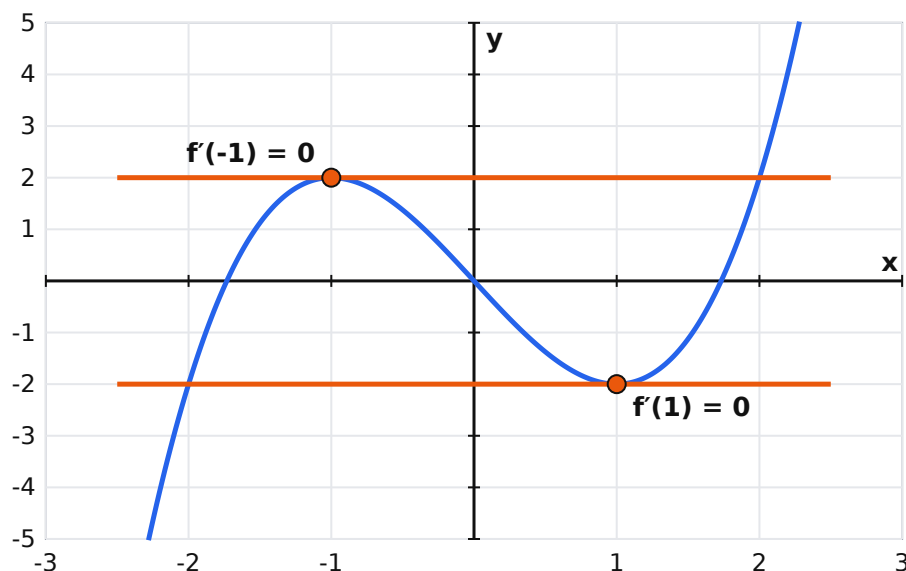
What is a good order of work?

1. Rewrite roots, fractions and negative powers as powers of x .
2. Differentiate each term separately, keeping constant multiples in front.
3. Simplify the derivative and substitute the given x -value if one is requested.
4. Include units when the function models distance, cost or another measured quantity.

Example. For $f(x) = \frac{3}{x} + 2\sqrt{x}$, rewrite $f(x) = 3x^{-1} + 2x^{1/2}$. Then $f'(x) = -3x^{-2} + x^{-1/2} = -\frac{3}{x^2} + \frac{1}{\sqrt{x}}$, and $f'(4) = -\frac{3}{16} + \frac{1}{2} = \frac{5}{16}$.

How do you find tangent and normal lines?

The tangent line at $x = a$ has slope $f'(a)$. The **normal line** is perpendicular to the tangent, so its slope is the negative reciprocal $-\frac{1}{f'(a)}$. Both pass through $(a, f(a))$.



Horizontal tangents occur where $f'(x) = 3x^2 - 3 = 0$.

Horizontal tangents occur where $f'(x) = 0$. For $f(x) = x^3 - 3x$, solve $3x^2 - 3 = 0$ to get $x = \pm 1$. The matching points are $(-1, 2)$ and $(1, -2)$, which are the peak and the valley of the graph.

Example. Find the tangent line to $f(x) = x^2 - 6x + 5$ at $x = 4$. First $f(4) = -3$. Next $f'(x) = 2x - 6$, so $f'(4) = 2$. The point-slope form $y + 3 = 2(x - 4)$ simplifies to $y = 2x - 11$. Check that the line and the curve both pass through $(4, -3)$ before you finish.

Questions about power, sum and basic function derivative rules

What is the power rule?

The power rule states that the derivative of x^n is $n x^{(n-1)}$. Multiply by the exponent, then reduce the exponent by one. It works for negative and fractional exponents too.

How do you differentiate a square root?

Rewrite the square root of x as $x^{(1/2)}$, apply the power rule to get $(1/2)x^{(-1/2)}$, and write the result as $1/(2 \sqrt{x})$.

What is the derivative of e^x ?

The derivative of e^x is e^x . It is the only basic exponential function that equals its own derivative.

How do you find where a graph has a horizontal tangent?

Compute $f'(x)$, set it equal to 0, and solve for x . Then evaluate f at each solution to get the points.

What is the difference between a tangent line and a normal line?

The tangent line touches the curve with slope $f'(a)$. The normal line is perpendicular to the tangent at the same point, so its slope is $-1/f'(a)$.