

AP Calculus AB - Power, Sum and Basic Function Derivative Rules - Answer Key

Common Core AP CALC AB 2.5, AP CALC AB 2.6, AP CALC AB 2.7 · maths-pdf.fr/en/

This answer key gives a full worked solution for every problem in the derivative rules worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 2.5, AP CALC AB 2.6, AP CALC AB 2.7**

Exercise 1 - Constant and power

- a. The derivative of a constant is 0.
- b. $g'(x) = 5x^4$. $5x^4$

Exercise 2 - Higher power

Apply the power rule: bring down 8 and subtract 1 from the exponent. Final answer: $8x^7$.

Exercise 3 - Polynomial

Differentiate term by term: $f'(x) = 12x^2 - 10x + 7$.

Final answer: $12x^2 - 10x + 7$.

Exercise 4 - Polynomial with a fraction

Differentiate term by term: $f'(x) = 12x^3 + x$.

Final answer: $12x^3 + x$.

Exercise 5 - Root and reciprocal

Rewrite with powers: $\sqrt{x} + \frac{5}{x^2}$.

Differentiate term by term: $f'(x) = -\frac{10}{x^3} + \frac{1}{2\sqrt{x}}$.

Final answer: $-\frac{10}{x^3} + \frac{1}{2\sqrt{x}}$.

Exercise 6 - Simplify first

Divide each term by x : $f(x) = x^4 - 3x$.

Then $f'(x) = 4x^3 - 3$. Final answer: $4x^3 - 3$.

Exercise 7 - Negative and fractional powers

Rewrite: $f(x) = 6x^{-3} - 2x^{1/3}$.

$f'(x) = -18x^{-4} - \frac{2}{3}x^{-2/3} = -\frac{18}{x^4} - \frac{2}{3x^{2/3}}$.

Final answer: $-\frac{18}{x^4} - \frac{2}{3\sqrt[3]{x^2}}$.

Exercise 8 - Sine and cosine

The term πx has derivative π because π is a constant.

$f'(x) = 4(-\sin x) + \frac{1}{2}\cos x + \pi$.

Final answer: $-4\sin x + \frac{1}{2}\cos x + \pi$.

Exercise 9 - Exponential and logarithm

$f'(x) = 5e^x - 4 \cdot \frac{1}{x}$.

Final answer: $5e^x - \frac{4}{x}$.

Exercise 10 - Mixed functions

The term π is a constant, so its derivative is 0.

$f'(x) = 2x + 2\cos x - e^x$. Final answer: $2x + 2\cos x - e^x$.

Exercise 11 - Evaluate a derivative

$$f'(x) = 3x^2 - 4x + 1.$$

$$f'(2) = 12 - 8 + 1 = 5. \text{ Final answer: } 5.$$

Exercise 12 - Horizontal tangents of a quartic

$$f'(x) = 4x^3 - 16x = 4x(x-2)(x+2). \text{ It is zero at } x = 0, \pm 2.$$

$$f(0) = 3 \text{ and } f(\pm 2) = 16 - 32 + 3 = -13.$$

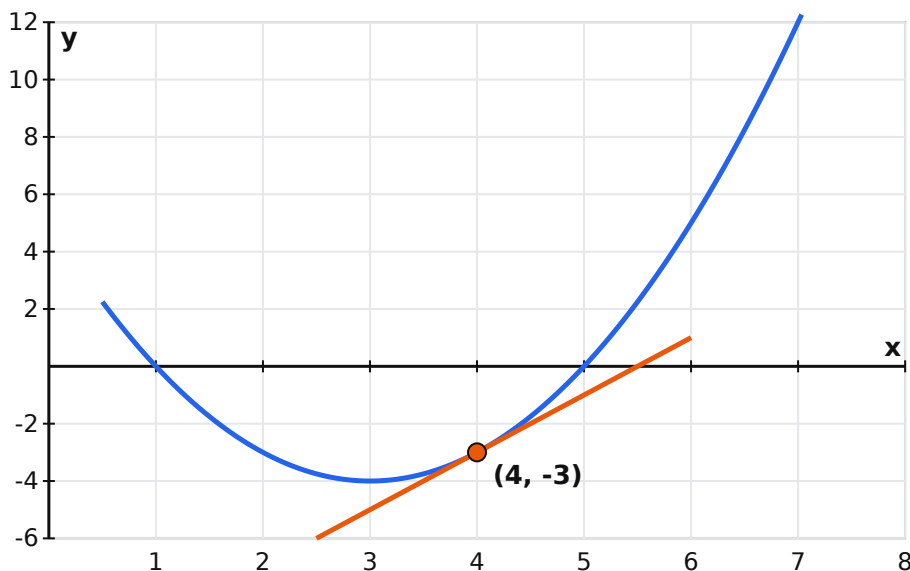
$$\text{Final answer: } (0, 3), (2, -13), (-2, -13).$$

Exercise 13 - Tangent line to a parabola

$$f(4) = 16 - 24 + 5 = -3 \text{ and } f'(x) = 2x - 6, \text{ so } f'(4) = 2.$$

$$\text{Point-slope: } y + 3 = 2(x - 4), \text{ so } y = 2x - 11.$$

$$\text{Final answer: } y = 2x - 11.$$

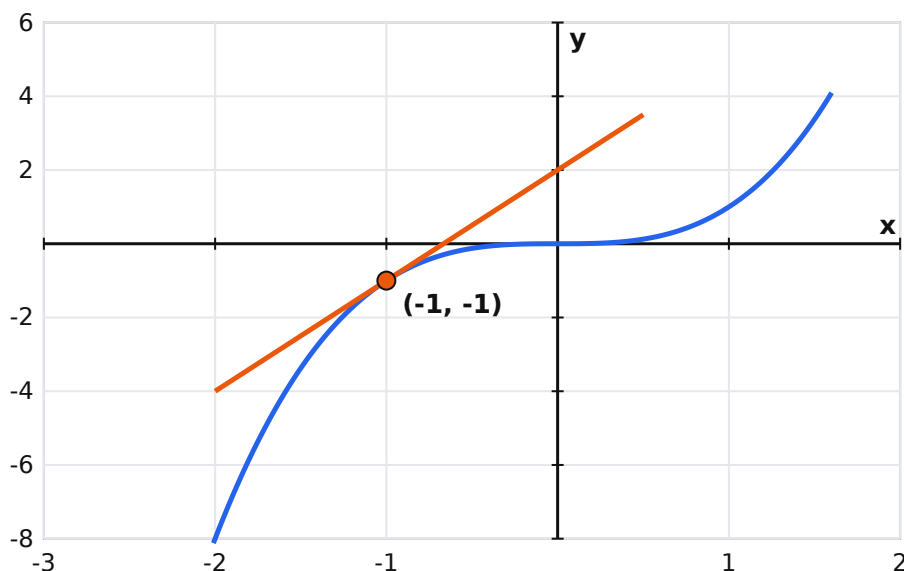


The tangent line at $x = 4$.

Exercise 14 - Tangent line to a cubic

$$f(-1) = -1 \text{ and } f'(-1) = 3(-1)^2 = 3.$$

$$y+1 = 3(x+1), \text{ so } y = 3x+2.$$



The tangent to $y = x^3$ at $x = -1$.

Exercise 15 - Normal line

$$f(4) = 2 \text{ and } f'(x) = \frac{1}{2\sqrt{x}}, \text{ so } f'(4) = \frac{1}{4}.$$

The normal slope is the negative reciprocal: -4 . So $y-2 = -4(x-4)$, which is $y = -4x+18$.

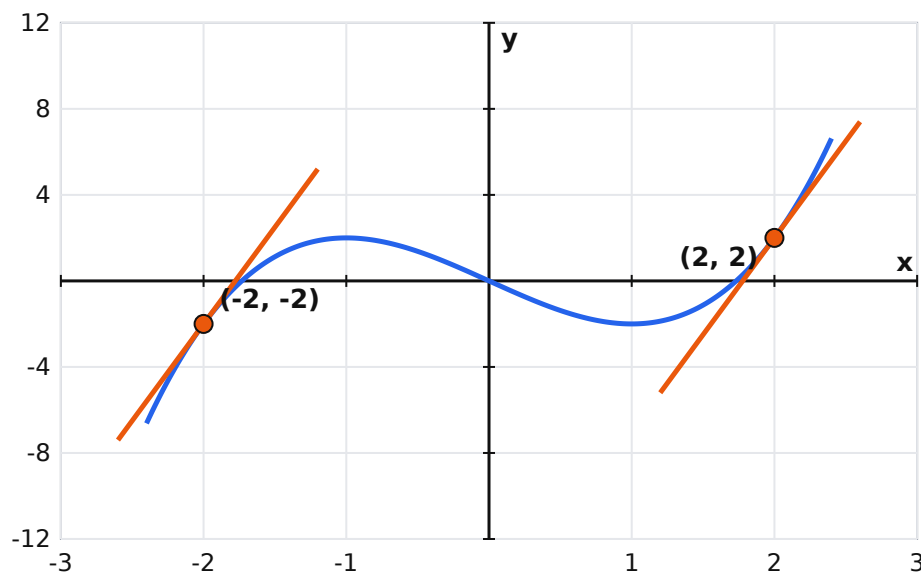
Final answer: $y = -4x+18$.

Exercise 16 - Parallel tangent lines

Parallel lines have equal slopes, so solve $f'(x) = 9$: $3x^2 - 3 = 9$, so $x^2 = 4$ and $x = \pm 2$.

$$f(2) = 8 - 6 = 2 \text{ and } f(-2) = -8 + 6 = -2.$$

Final answer: $(2, 2)$ **and** $(-2, -2)$.



Two parallel tangents of slope 9.

Exercise 17 - Tangent to the exponential

$$f(0)=1 \text{ and } f'(0)=e^0=1.$$

Final answer: $y = x+1$.

Exercise 18 - Tangent to the logarithm

$$f(e)=1 \text{ and } f'(e)=\frac{1}{e}.$$

$$y-1=\frac{1}{e}(x-e) \text{ simplifies to } y=\frac{x}{e}.$$

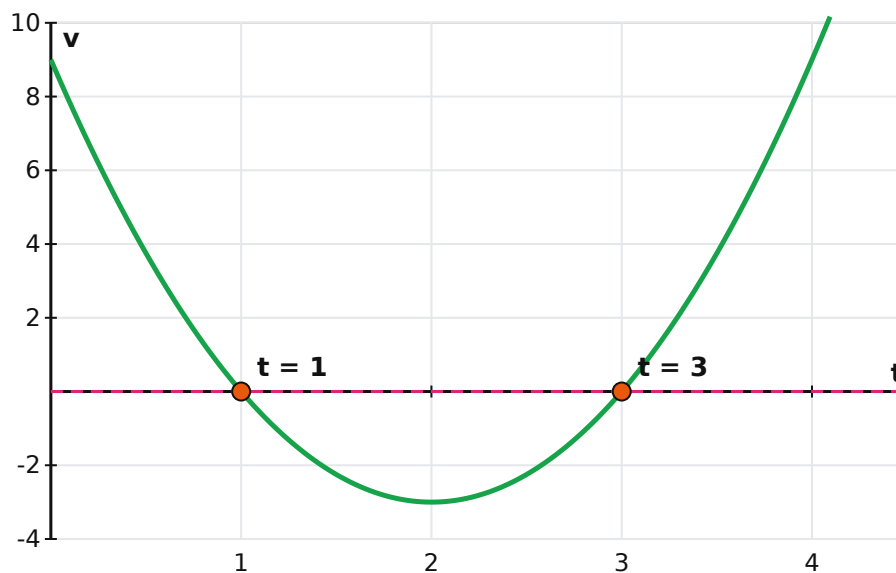
Final answer: $y=\frac{x}{e}$.

Exercise 19 - Velocity of a particle

$$v(t)=s'(t)=3t^2-12t+9=3(t-1)(t-3).$$

At rest when $v(t)=0$: $t=1$ or $t=3$.

Final answer: $v(t)=3t^2-12t+9$ ft/s; at rest at $t=1$ s and $t=3$ s.



Velocity is zero at $t = 1$ and $t = 3$.

Exercise 20 - Maximum profit

$P'(x) = -2x + 50$. Set it equal to 0: $x = 25$.

$P(25) = -625 + 1250 - 200 = 425$.

Final answer: horizontal tangent at $x = 25$ **candles**, where the profit is **\\$425**.

Exercise 21 - Finding coefficients

Point: $f(1) = a + b = 3$. Slope: $f'(x) = 2ax + b$, so $f'(1) = 2a + b = 2$.

Subtract the first equation from the second: $a = -1$, then $b = 4$.

Final answer: $a = -1$, $b = 4$.

Exercise 22 - Tangent through an outside point

The slope is $2a$, so the tangent is $y - a^2 = 2a(x - a)$. At $x = 0$: $y = a^2 - 2a^2 = -a^2$.

Set $-a^2 = -4$: $a = \pm 2$.

Final answer: $a = 2$ or $a = -2$.