

AP Calculus AB - Differential Equations and Slope Fields - Lesson

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A differential equation relates a function to its derivative. A slope field shows dy/dx at many points, and Euler's method approximates a solution with tangent line steps. Separable equations are solved by separating variables, integrating, and using an initial condition. If $dy/dt = ky$, then $y = y_0 e^{(kt)}$.

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Key facts

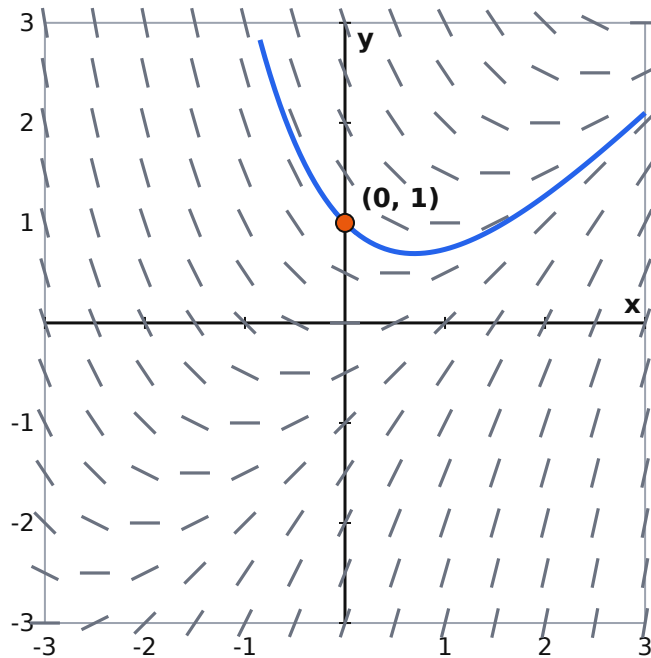
- Verify a solution by substituting y and y' into the equation.
- A slope field shows dy/dx at grid points; solution curves follow the segments.
- Euler: $y_{\text{new}} = y_{\text{old}} + h \cdot (dy/dx \text{ at the old point})$, $x_{\text{new}} = x_{\text{old}} + h$.
- $dy/dt = ky$ has the general solution $y = Ce^{(kt)}$; growth if k is positive, decay if k is negative.

What is a differential equation?

A **differential equation** relates a function to its derivative, such as $\frac{dy}{dx} = x - y$. A *solution* is a function that makes the equation true. You can verify a proposed solution by substituting it and its derivative into the equation. For example, $y = e^{3x}$ solves $\frac{dy}{dx} = 3y$ because $\frac{d}{dx}e^{3x} = 3e^{3x}$.

What is a slope field?

A slope field draws, at many points (x, y) , a short segment with slope $\frac{dy}{dx}$ evaluated there. Solution curves follow the segments like a river following arrows.

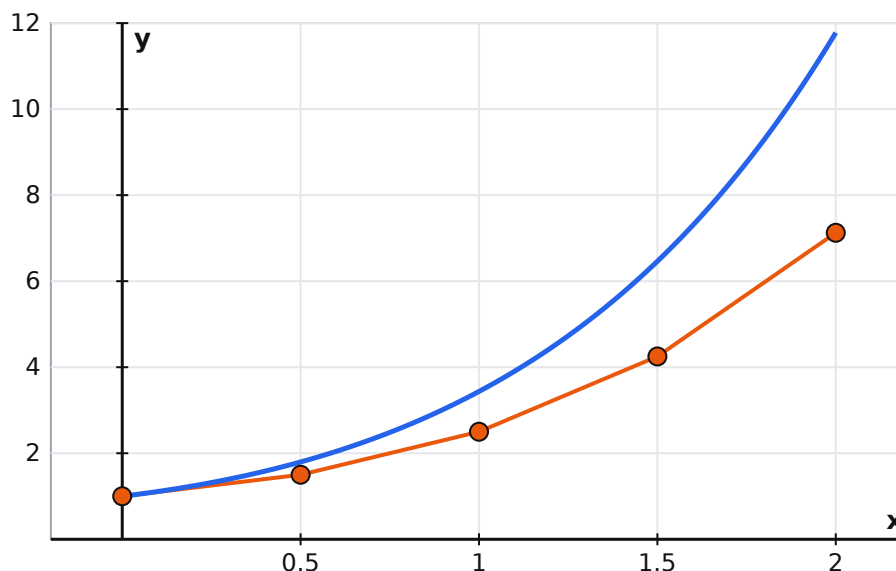


Slope field of $dy/dx = x - y$ with the solution through $(0, 1)$.

- Where $\frac{dy}{dx} = 0$ the segments are horizontal.
- If the formula depends only on x , segments in a vertical line are parallel.
- If it depends only on y , segments in a horizontal line are parallel.

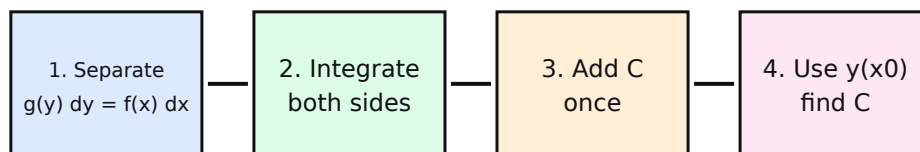
What is Euler's method?

Euler's method approximates a solution by following tangent lines. Starting at (x_0, y_0) with step h : $y_{n+1} = y_n + h \cdot \frac{dy}{dx}(x_n, y_n)$ and $x_{n+1} = x_n + h$. For $\frac{dy}{dx} = x + y$ with $y(0) = 1$ and $h = 0.5$, $y_1 = 1 + 0.5(0 + 1) = 1.5$.



Euler's method (orange) follows tangent segments and lags behind the true solution (blue).

How do you solve by separation of variables?



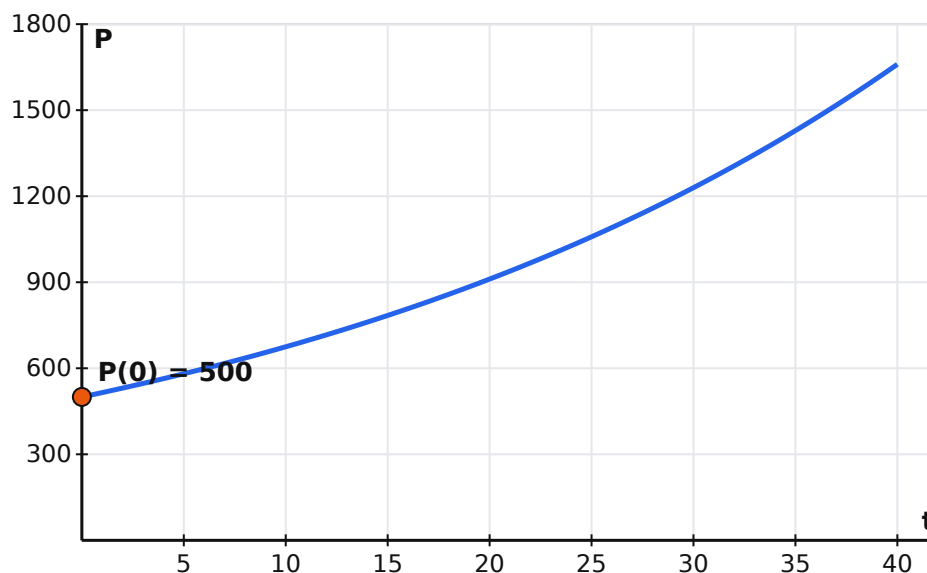
Four steps for separable differential equations.

Example. Solve $\frac{dy}{dx} = \frac{x}{y}$ with $y(0) = 2$. Separate: $y \, dy = x \, dx$. Integrate: $\frac{y^2}{2} = \frac{x^2}{2} + C$. Use $y(0) = 2$ to get $C = 2$, so $y^2 = x^2 + 4$ and $y = \sqrt{x^2 + 4}$ (positive because $y(0) > 0$).

To check a separated solution, differentiate it. For $y = \sqrt{x^2 + 4}$, $\frac{dy}{dx} = \frac{x}{\sqrt{x^2 + 4}} = \frac{x}{y}$, which is the original equation, and $y(0) = \sqrt{4} = 2$ matches the initial condition. Always report the interval where the solution is valid: for $\frac{dy}{dx} = y^2$ with $y(0) = 1$ the solution $y = \frac{1}{1-x}$ exists only for $x < 1$.

What are exponential growth and decay models?

If a quantity changes at a rate proportional to itself, $\frac{dy}{dt} = ky$, then $y = y_0 e^{kt}$. With $k > 0$ this is growth; with $k < 0$, decay. For a population $\frac{dP}{dt} = 0.03P$ with $P(0) = 500$, $P = 500e^{0.03t}$, so $P(10) = 500e^{0.3} \approx 675$.



Exponential growth: $dP/dt = 0.03P$ gives $P = 500e^{(0.03t)}$.

Two features help you recognize exponential models. First, the growth rate $\frac{dP}{dt}$ is proportional to P , so a larger population grows faster. Second, the time to double does not depend on the starting size: the doubling time is always $\frac{\ln 2}{k}$. For decay with half-life T , the constant is $k = \frac{\ln 2}{T}$.

Questions about differential equations and slope fields

What is a differential equation?

It is an equation that involves a function and its derivatives. A solution is a function that satisfies the equation, often with an initial condition that picks out one particular solution.

How do you read a slope field?

Each short segment shows the slope of a solution curve at that point. Follow the segments to sketch a solution through a given starting point.

How does Euler's method work?

Start at the initial point, compute the slope from the differential equation, move horizontally by the step h along the tangent line, and repeat from the new point.

What is separation of variables?

It rewrites $dy/dx = f(x)g(y)$ as $dy/g(y) = f(x) dx$ so that each side can be integrated separately. Remember to add the constant C and use the initial condition.

What is the solution of $dy/dt = ky$?

The solution is $y = Ce^{(kt)}$, where $C = y(0)$. A positive k gives exponential growth and a negative k gives exponential decay.