

AP Calculus AB - Differential Equations and Slope Fields - Answer Key

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This answer key gives a full worked solution for every problem in the differential equations worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 7.1, AP CALC AB 7.2, AP CALC AB 7.3, AP CALC AB 7.4, AP CALC AB 7.5, AP CALC AB 7.6, AP CALC AB 7.7**

Exercise 1 - Verify a solution

- a. $\frac{d}{dx}(x^2 + C) = 2x$, which equals the right side.
- b. $\frac{d}{dx}(3e^{2x}) = 6e^{2x}$ and $2y = 2 \cdot 3e^{2x} = 6e^{2x}$. They match.

Final answer: both functions solve their equations.

Exercise 2 - Verify a second-order solution

$$y' = e^x + xe^x = (1+x)e^x \text{ and } y'' = (2+x)e^x.$$

$$y'' - 2y' + y = (2+x)e^x - 2(1+x)e^x + xe^x = e^x(2+x-2-2x+x) = 0.$$

Final answer: verified.

Exercise 3 - Reading a slope field

$x - y$ at (1,0): 1. At (0,2): -2. At (2,2): 0.

Segments are horizontal where $x - y = 0$, that is along the line $y = x$.

Final answer: 1, -2, 0; horizontal along $y = x$.

Exercise 4 - Matching an equation

At $y=0$ the slope is 0 (horizontal segments). For $y>0$ the slope is positive and increases with y ; for $y<0$ it is negative.

The slope depends only on y , so segments along a horizontal line are parallel and they change as you move up or down.

Final answer: depends only on y ; zero at $y=0$.

Exercise 5 - Euler, two steps

Step 1: slope at $(0,1)$ is 1, so $y_1 = 1 + 0.1(1) = 1.1$ at $x = 0.1$.

Step 2: slope at $(0.1, 1.1)$ is 1.2, so $y_2 = 1.1 + 0.1(1.2) = 1.22$.

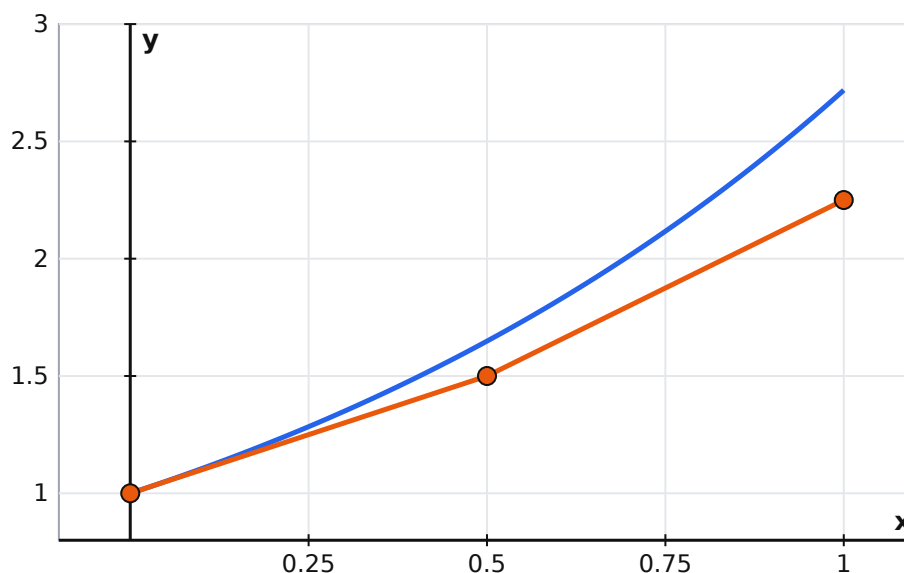
Final answer: $y(0.2) \approx 1.22$.

Exercise 6 - Euler for y' equals y

$y_1 = 1 + 0.5(1) = 1.5$, $y_2 = 1.5 + 0.5(1.5) = 2.25$.

The exact solution is $y = e^x$, so $y(1) = e \approx 2.718$. Euler underestimates because the solution is concave up.

Final answer: $y(1) \approx 2.25$.



Euler (orange) underestimates e^x (blue).

Exercise 7 - Euler with a decreasing solution

$$y_1 = 2 + 0.25(0 - 2) = 1.5 \text{ at } x = 0.25.$$

$$y_2 = 1.5 + 0.25(0.25 - 1.5) = 1.5 - 0.3125 = 1.1875.$$

Final answer: $y(0.5) \approx 1.1875$.

Exercise 8 - Separable: exponential

Separate: $\frac{dy}{y} = 2x \, dx$. Integrate: $\ln y = x^2 + C$.

Exponentiate: $y = e^C e^{x^2} = A e^{x^2}$.

Final answer: $y = A e^{x^2}$.

Exercise 9 - Separable with a trigonometric factor

Separate: $\frac{dy}{y} = \cos x \, dx$. Integrate: $\ln|y| = \sin x + C$, so $y = A e^{\sin x}$.

$$y\left(\frac{\pi}{2}\right) = A e = e \text{ gives } A = 1.$$

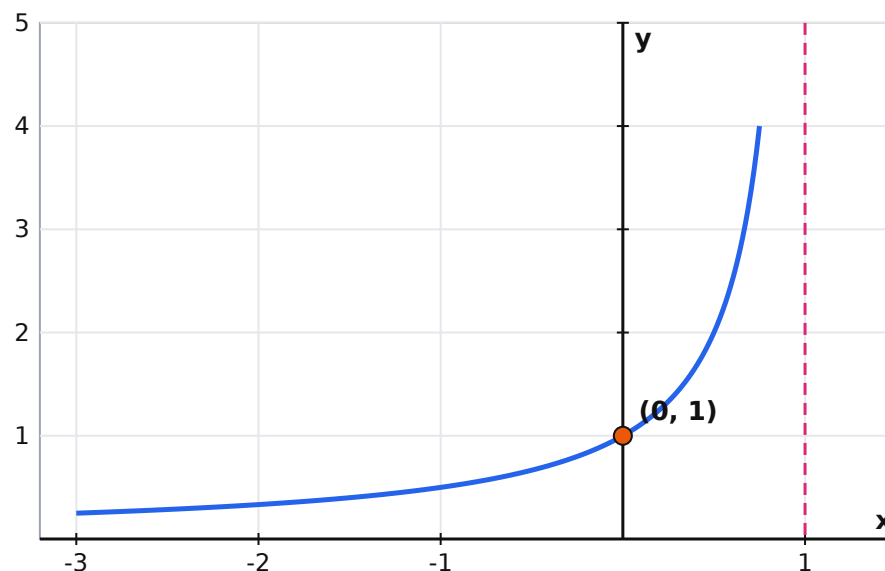
Final answer: $y = e^{\sin x}$.

Exercise 10 - Separable: y squared

Separate: $\frac{dy}{y^2} = dx$. Integrate: $-\frac{1}{y} = x + C$. $y(0) = 1$ gives $C = -1$.

$$-\frac{1}{y} = x - 1, \text{ so } y = \frac{1}{1-x}, \text{ valid for } x < 1.$$

Final answer: $y = \frac{1}{1-x}, x < 1$.

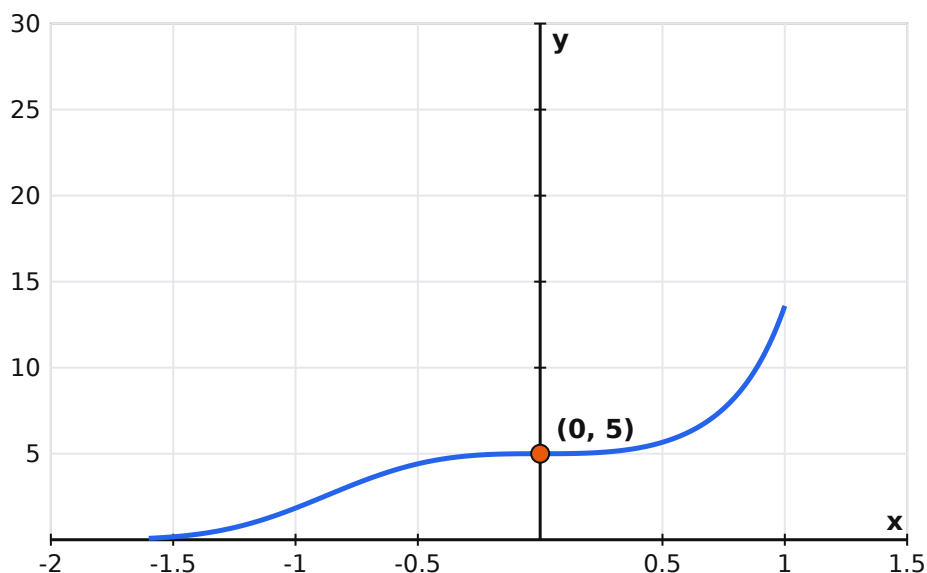


The solution blows up at $x = 1$.

Exercise 11 - Separable: x squared times y

$\frac{dy}{y} = 3x^2 dx$, so $\ln|y| = x^3 + C$ and $y = Ae^{x^3}$. $y(0) = A = 5$.

Final answer: $y = 5e^{x^3}$.



The solution $y = 5e^{x^3}$.

Exercise 12 - Separable: a clever substitution

$e^{x-y} = \frac{e^x}{e^y}$, so $e^y dy = e^x dx$ and $e^y = e^x + C$.

$y(0)=0$ gives $1=1+C$, so $C=0$, $e^y=e^x$ and $y=x$.

Final answer: $y=x$.

Exercise 13 - Separable: y over x

$\frac{dy}{y} = \frac{dx}{x}$, so $\ln y = \ln x + C$ and $y = Ax$. $y(1) = A = 3$.

Final answer: $y=3x$.

Exercise 14 - Decay equation

$y = Ce^{-2x}$ and $y(0)=4$ gives $C=4$.

$y(1) = 4e^{-2} \approx 0.541$.

Final answer: $y = 4e^{-2x}$; $y(1) \approx 0.541$.

Exercise 15 - Population growth

$P = P_0 e^{kt} = 500e^{0.03t}$.

$P(10) = 500e^{0.3} \approx 674.9$.

Final answer: $P = 500e^{0.03t}$; $P(10) \approx 674.9$.

Exercise 16 - Doubling time

$N = N_0 e^{kt}$. Doubling at $t=5$: $e^{5k} = 2$.

$k = \frac{\ln 2}{5} \approx 0.1386$ per hour.

Final answer: $k = \frac{\ln 2}{5} \approx 0.139$.

Exercise 17 - Half-life

$e^{-10k} = \frac{1}{2}$ gives $k = \frac{\ln 2}{10}$.

After 30 days: $e^{-30k} = (e^{-10k})^3 = \frac{1}{8}$.

Final answer: $\frac{1}{8}$ (12.5%).

Exercise 18 - Newton cooling

Let $u = T - 70$: $\frac{du}{dt} = -0.1u$, so $u = 120e^{-0.1t}$ (since $u(0) = 120$).

$T(t) = 70 + 120e^{-0.1t}$ and $T(10) = 70 + \frac{120}{e} \approx 114.1$.

Final answer: $T(t) = 70 + 120e^{-0.1t}$; $T(10) \approx 114.1^\circ\text{F}$.

Exercise 19 - Continuous interest

$A = 2000e^{0.05t}$, so $A(10) = 2000e^{0.5} \approx 3297.4$.

Final answer: \$3297.44.

Exercise 20 - General and particular solution

Separate: $\frac{dy}{y-1} = x \, dx$. Integrate: $\ln|y-1| = \frac{x^2}{2} + C$, so $y = 1 + Ae^{x^2/2}$.

$y(0) = 1 + A = 3$ gives $A = 2$.

Final answer: $y = 1 + 2e^{x^2/2}$.

Exercise 21 - General exponential model

Separate: $\frac{dy}{y} = k \, dt$. Integrate: $\ln|y| = kt + C$.

Exponentiate: $y = \pm e^{C} e^{kt} = Ae^{kt}$. At $t = 0$, $y(0) = A$.

Final answer: $y = y_0 e^{kt}$.