

AP Calculus AB - Differential Equations and Slope Fields - Worksheet

Common Core AP CALC AB 7.1, AP CALC AB 7.2, AP CALC AB 7.3, AP CALC AB 7.4, AP CALC AB 7.5, AP CALC AB 7.6, AP CALC AB 7.7 · maths-pdf.fr/en/

Name: _____ Class: _____ Date: ____/____/____

These 21 problems practice differential equations, slope fields, Euler's method, separation of variables and exponential growth and decay (AP Calculus AB, topics 7.1 to 7.7). The early problems build the basic skill, and each one uses a different function or situation so that no two feel alike.

Later problems use Euler's method, initial value problems, Newton cooling and real situations such as population growth and continuous interest. Work in pencil, show every step, and check the answer key only after a full attempt.

AP Calculus AB · Common Core State Standards: **AP CALC AB 7.1, AP CALC AB 7.2, AP CALC AB 7.3, AP CALC AB 7.4, AP CALC AB 7.5, AP CALC AB 7.6, AP CALC AB 7.7**

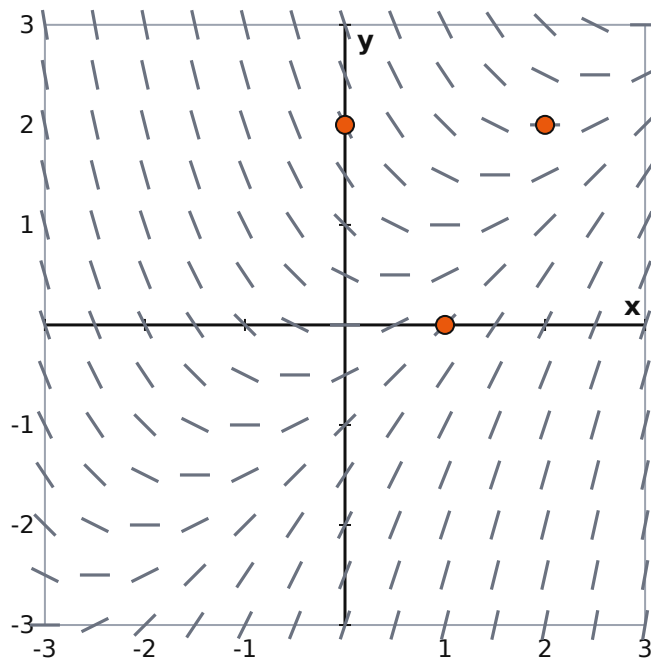
Exercise 1 - Verify a solution

Verify that $y = x^2 + C$ solves $\frac{dy}{dx} = 2x$, and that $y = 3e^{2x}$ solves $\frac{dy}{dx} = 2y$.

Exercise 2 - Verify a second-order solution

Show that $y = xe^x$ is a solution of $y'' - 2y' + y = 0$.

Exercise 3 - Reading a slope field



Slope field of $dy/dx = x - y$.

The slope field is for $\frac{dy}{dx} = x - y$. Find the slope at $(1,0)$, $(0,2)$ and $(2,2)$. Where are the segments horizontal?

Exercise 4 - Matching an equation

For $\frac{dy}{dx} = y$, describe the slope field along the line $y = 0$, for $y > 0$ and for $y < 0$. Is the slope the same along a vertical line?

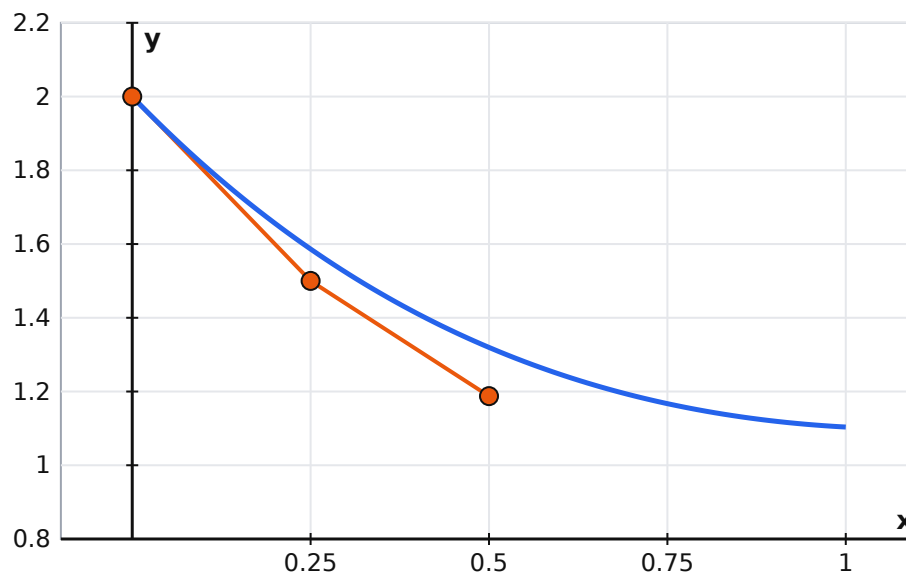
Exercise 5 - Euler, two steps

Use Euler's method with step $h = 0.1$ to estimate $y(0.2)$ for $\frac{dy}{dx} = x + y$, $y(0) = 1$.

Exercise 6 - Euler for y prime equals y

Use Euler's method with $h = 0.5$ to estimate $y(1)$ for $\frac{dy}{dx} = y$, $y(0) = 1$. Compare with the exact value.

Exercise 7 - Euler with a decreasing solution



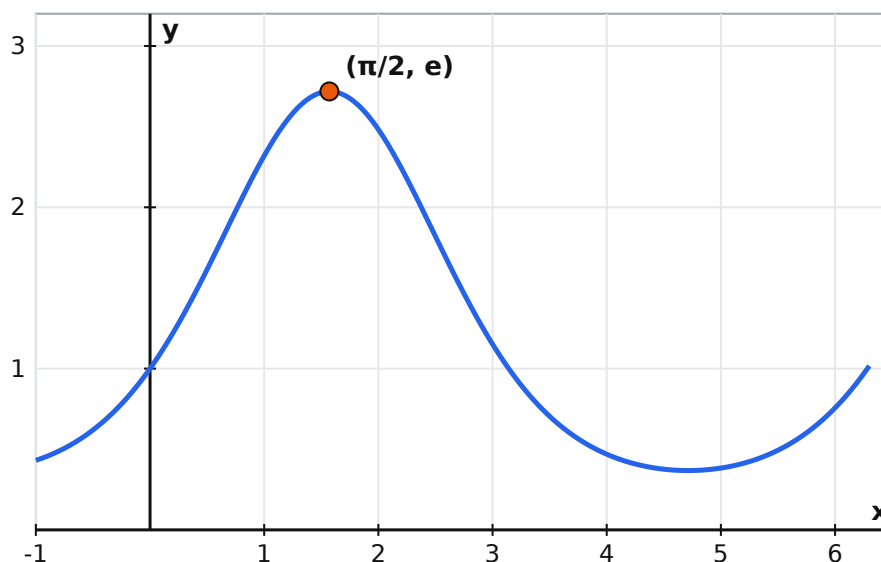
Euler steps for $dy/dx = x - y$, $y(0) = 2$.

Use Euler's method with $h = 0.25$ to estimate $y(0.5)$ for $\frac{dy}{dx} = x - y$, $y(0) = 2$.

Exercise 8 - Separable: exponential

Solve $\frac{dy}{dx} = 2xy$ for $y > 0$.

Exercise 9 - Separable with a trigonometric factor



The solution through $(\pi/2, e)$.

Solve $\frac{dy}{dx} = y \cos x$ with $y\left(\frac{\pi}{2}\right) = e$.

Exercise 10 - Separable: y squared

Solve $\frac{dy}{dx} = y^2$ with $y(0) = 1$, and state the interval on which the solution exists.

Exercise 11 - Separable: x squared times y

Solve $\frac{dy}{dx} = 3x^2y$ with $y(0) = 5$.

Exercise 12 - Separable: a clever substitution

Solve $\frac{dy}{dx} = e^{x-y}$ with $y(0) = 0$.

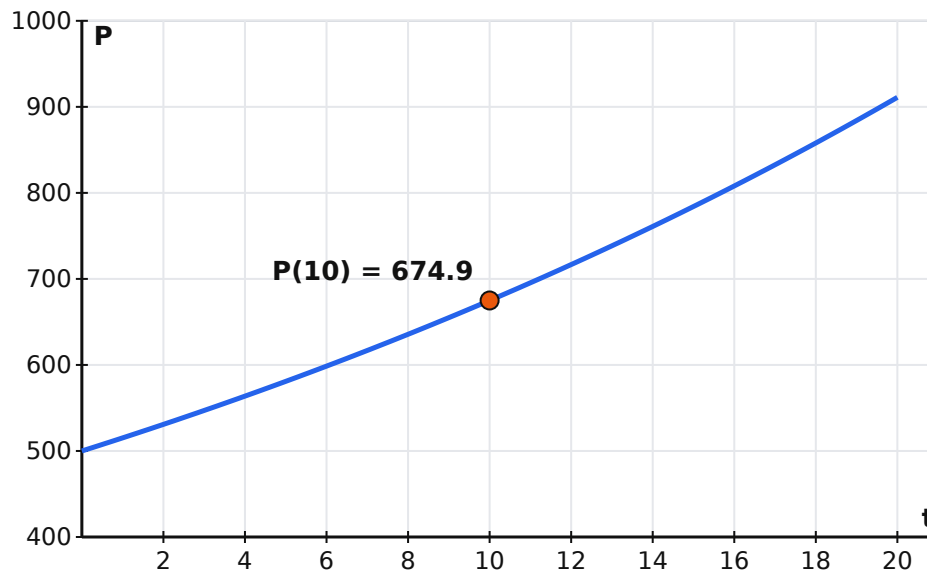
Exercise 13 - Separable: y over x

Solve $\frac{dy}{dx} = \frac{y}{x}$ with $y(1) = 3$ for $x > 0$.

Exercise 14 - Decay equation

Solve $\frac{dy}{dx} = -2y$ with $y(0) = 4$, and find $y(1)$.

Exercise 15 - Population growth



A town's population over 20 years.

A town's population grows at a rate proportional to its size: $\frac{dP}{dt} = 0.03P$, $P(0) = 500$ with t in years. Find $P(t)$ and $P(10)$.

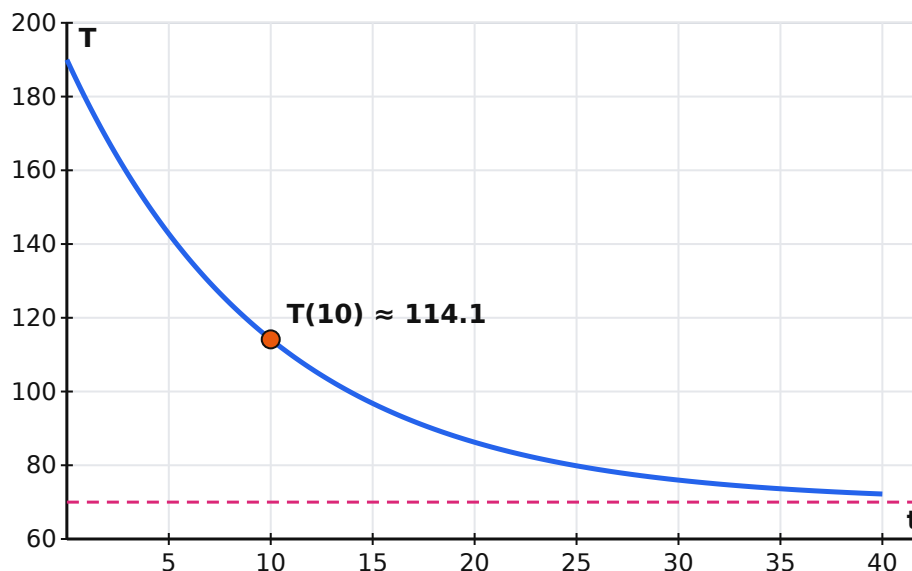
Exercise 16 - Doubling time

A bacteria culture doubles every 5 hours and satisfies $\frac{dN}{dt} = kN$. Find k .

Exercise 17 - Half-life

A medicine decays so that $\frac{dA}{dt} = -kA$ with a half-life of 10 days. What fraction remains after 30 days?

Exercise 18 - Newton cooling



Cooling toward 70 °F.

A cup of tea at 190 °F cools in a 70 °F room according to $\frac{dT}{dt} = -0.1(T - 70)$. Find $T(t)$ and $T(10)$.

Exercise 19 - Continuous interest

An account earns continuously compounded interest at $\frac{dA}{dt} = 0.05A$ with $A(0) = 2000$ dollars. Find $A(10)$.

Exercise 20 - General and particular solution

Solve $\frac{dy}{dx} = x(y-1)$, then find the solution with $y(0) = 3$.

Exercise 21 - General exponential model

Show that every solution of $\frac{dy}{dt} = ky$ has the form $y = y_0 e^{kt}$ where $y_0 = y(0)$.