

AP Calculus AB - Differential Equations and Slope Fields - Test Answer Key

Common Core AP CALC AB 7.1, AP CALC AB 7.2, AP CALC AB 7.3, AP CALC AB 7.4, AP CALC AB 7.5, AP CALC AB 7.6, AP CALC AB 7.7 · maths-pdf.fr/en/

Use this answer key to score the differential equations test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

AP Calculus AB · Common Core State Standards: **AP CALC AB 7.1, AP CALC AB 7.2, AP CALC AB 7.3, AP CALC AB 7.4, AP CALC AB 7.5, AP CALC AB 7.6, AP CALC AB 7.7**

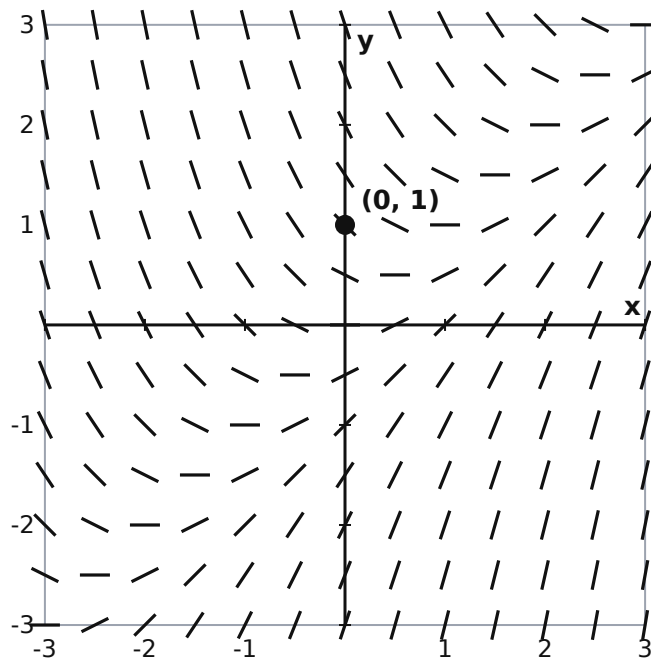
Question 1 - Verify a solution [2 pts]

$$\frac{dy}{dx} = 4(-3)e^{-3x} = -12e^{-3x} \text{ and } -3y = -3 \cdot 4e^{-3x} = -12e^{-3x}. \text{ They are equal.}$$

Scoring: 1 point for the derivative; 1 point for showing both sides are equal.

Question 2 - Slope field analysis [3 pts]

- a. $2 - (-1) = 3$.
- b. Where $x = y$, along the line $y = x$.
- c. At $(0,1)$ the slope is $0 - 1 = -1$, so the solution is decreasing there.



Slope field for $dy/dx = x - y$.

Scoring: 1 point per part.

Question 3 - Euler's method [2 pts]

Step 1: slope at $(0,1)$ is -1 , so $y_1 = 1 + 0.5(-1) = 0.5$ at $x = 0.5$.

Step 2: slope at $(0.5, 0.5)$ is $2(0.5) - 0.5 = 0.5$, so $y_2 = 0.5 + 0.5(0.5) = 0.75$.

Final answer: $y(1) \approx 0.75$.

Scoring: 1 point per step.

Question 4 - Separation of variables [3 pts]

Separate: $y \, dy = 2x \, dx$. Integrate: $\frac{y^2}{2} = x^2 + C$.

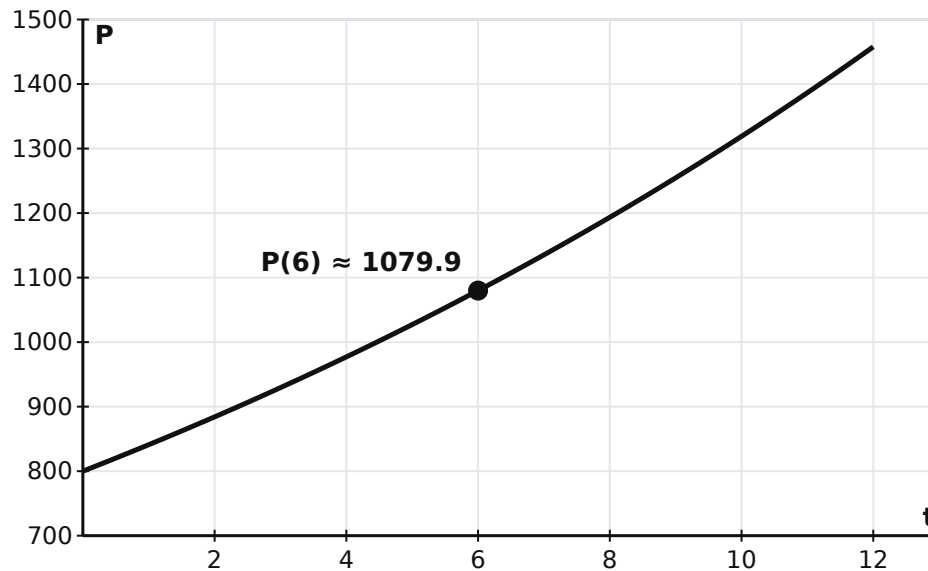
$y(0) = 3$ gives $C = \frac{9}{2}$, so $y^2 = 2x^2 + 9$.

Final answer: $y = \sqrt{2x^2 + 9}$.

Scoring: 1 point for separating; 1 point for integrating; 1 point for the particular solution.

Question 5 - Exponential growth [3 pts]

- a. $P(t) = 800e^{0.05t}$.
b. $P(6) = 800e^{0.3} \approx 1079.9$.
c. $800e^{0.05t} = 1600$ gives $t = \frac{\ln 2}{0.05} \approx 13.9$ years.



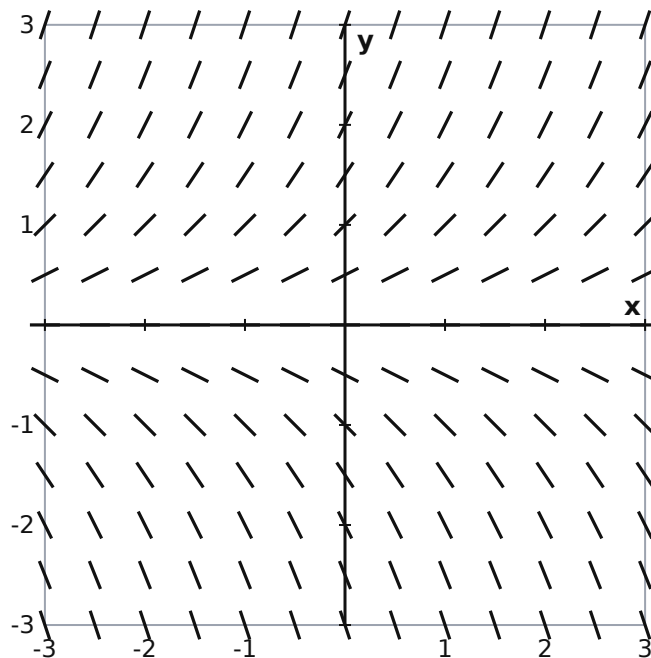
Fish population in a lake.

Scoring: 1 point per part.

Question 6 - Matching a slope field [2 pts]

The slope is 0 along $y=0$ and has the same sign as y , and it does not change with x .

Final answer: $\frac{dy}{dx} = y$.



A slope field.

Scoring: 1 point for the match; 1 point for the explanation.

Question 7 - Cooling [3 pts]

- $\frac{du}{dt} = -0.2u$, $u = Ce^{-0.2t}$, $u(0) = 5 - 25 = -20$, so $T(t) = 25 - 20e^{-0.2t}$.
- $T(5) = 25 - 20e^{-1} \approx 17.64$ °C.

Scoring: (a) 1 point for the substitution and equation; 1 point for the solution; (b) 1 point.

Question 8 - Half-life [2 pts]

$$e^{-8k} = \frac{1}{2}. \text{ After 24 days: } \left(\frac{1}{2}\right)^3 = \frac{1}{8}. \text{ After 12 days: } e^{-12k} = \left(\frac{1}{2}\right)^{3/2} = \frac{1}{2\sqrt{2}} \approx 0.354.$$

Scoring: 1 point for each fraction.

Total

20 points