

AP Calculus AB - Differential Equations and Slope Fields - Test

Common Core AP CALC AB 7.1, AP CALC AB 7.2, AP CALC AB 7.3, AP CALC AB 7.4, AP CALC AB 7.5, AP CALC AB 7.6, AP CALC AB 7.7 · maths-pdf.fr/en/

Name: _____ Class: _____ Date: ____/____/____

This test checks differential equations, slope fields, Euler's method, separation of variables and exponential growth and decay (AP Calculus AB, 7.1 to 7.7). It has 8 questions worth 20 points in total. Allow 45 minutes, work without notes, and show all work, since justification earns points on the AP exam.

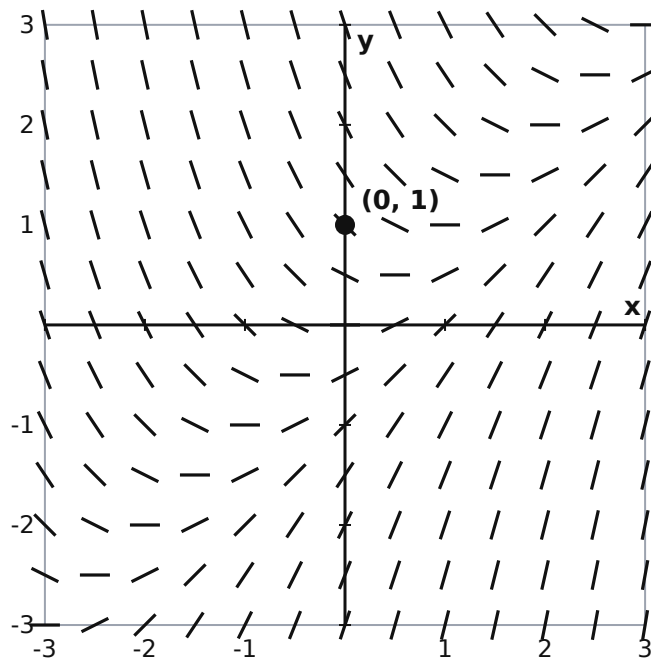
About this test

- Grade: AP Calculus AB
- Chapter: Differential Equations and Slope Fields
- Standards: AP CALC AB 7.1, AP CALC AB 7.2, AP CALC AB 7.3, AP CALC AB 7.4, AP CALC AB 7.5, AP CALC AB 7.6, AP CALC AB 7.7
- Time: 45 minutes
- Total: 20 points

Question 1 - Verify a solution [2 pts]

Show that $y = 4e^{-3x}$ is a solution of $\frac{dy}{dx} = -3y$.

Question 2 - Slope field analysis [3 pts]



Slope field for $dy/dx = x - y$.

The slope field is for $\frac{dy}{dx} = x - y$. (a) Find the slope at $(2, -1)$. (1 point) (b) Where are the segments horizontal? (1 point) (c) Describe the solution through $(0, 1)$ near that point (rising or falling). (1 point)

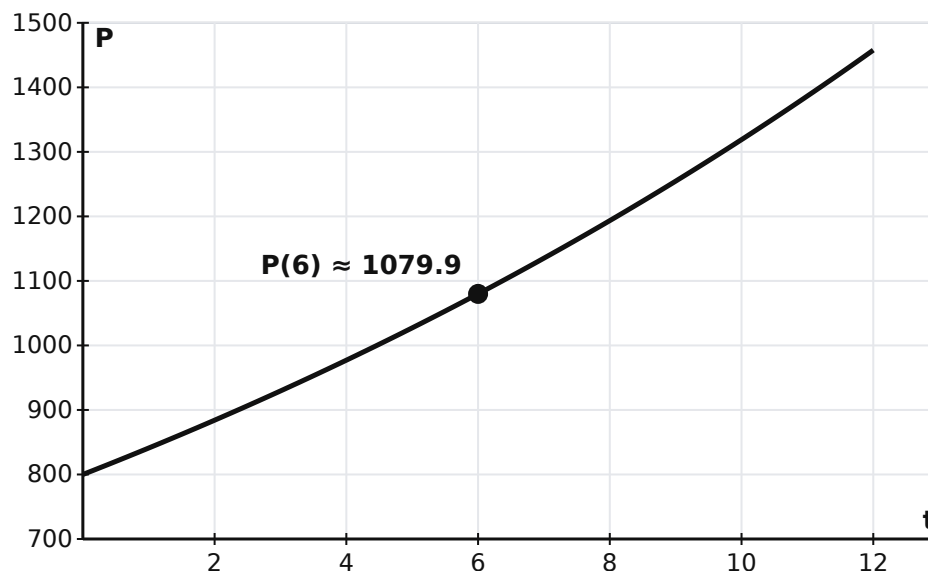
Question 3 - Euler's method [2 pts]

Use Euler's method with $h = 0.5$ to estimate $y(1)$ for $\frac{dy}{dx} = 2x - y$, $y(0) = 1$.

Question 4 - Separation of variables [3 pts]

Solve $\frac{dy}{dx} = \frac{2x}{y}$ with $y(0) = 3$.

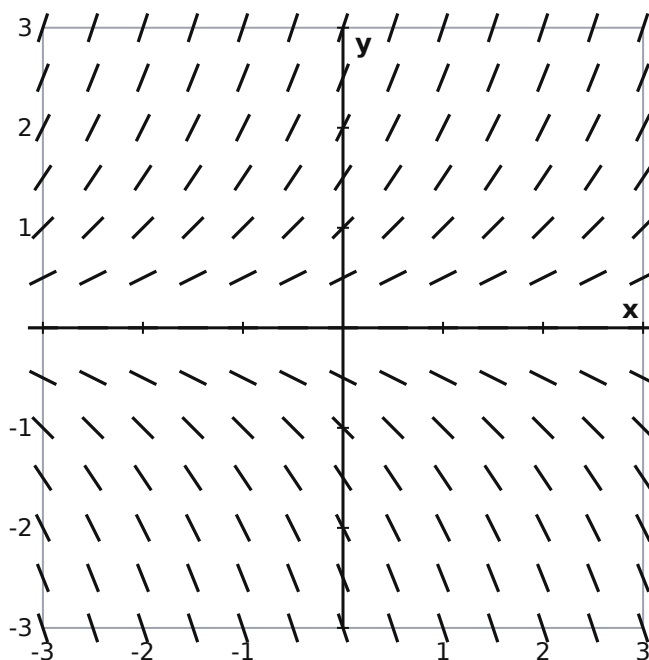
Question 5 - Exponential growth [3 pts]



Fish population in a lake.

A fish population satisfies $\frac{dP}{dt} = 0.05P$ with $P(0) = 800$ and t in years. (a) Find $P(t)$. (1 point) (b) Find $P(6)$. (1 point) (c) When does the population reach 1600? (1 point)

Question 6 - Matching a slope field [2 pts]



A slope field.

The slope field depends only on y : horizontal segments at $y=0$, positive slopes for $y>0$ and negative slopes for $y<0$. Which equation matches: $\frac{dy}{dx} = x$, $\frac{dy}{dx} = y$ or $\frac{dy}{dx} = x+y$? Explain.

Question 7 - Cooling [3 pts]

A cold drink at 5 °C is placed in a 25 °C room and warms according to $\frac{dT}{dt} = -0.2(T - 25)$ with t in minutes. (a) Let $u = T - 25$ and solve for $T(t)$. (2 points) (b) Find $T(5)$. (1 point)

Question 8 - Half-life [2 pts]

A radioactive sample decays by $\frac{dN}{dt} = -kN$ with a half-life of 8 days. What fraction of the sample remains after 24 days? After 12 days (exact)?