

AP Calculus AB - Increasing, Decreasing and the First Derivative Test - Lesson

Common Core AP CALC AB 5.4, AP CALC AB 5.5, AP CALC AB 5.6 · maths-pdf.fr/en/

A function increases where its derivative is positive and decreases where its derivative is negative. The First Derivative Test uses a sign chart of f' : a change from positive to negative at a critical number gives a relative maximum, and a change from negative to positive gives a relative minimum. If f' does not change sign, there is no extremum.

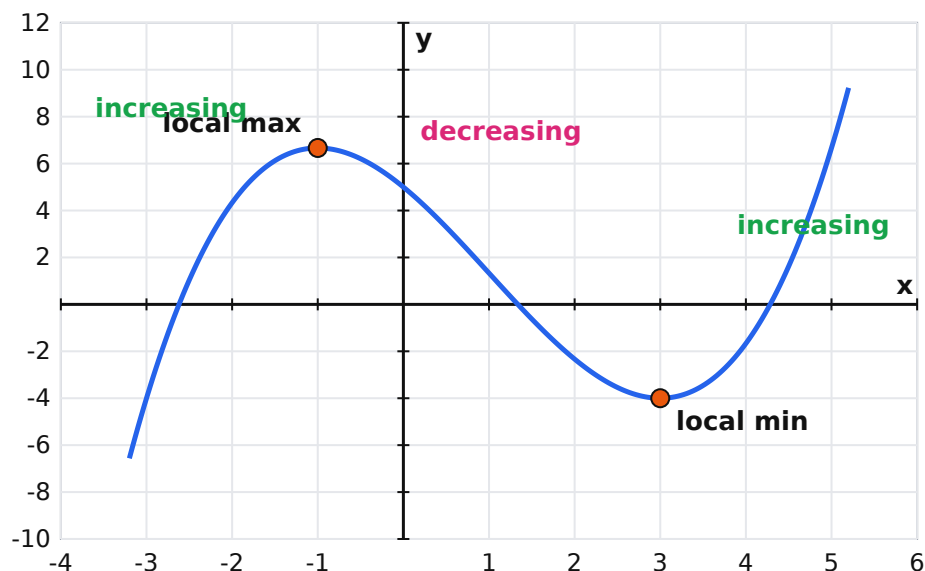
AP Calculus AB · Common Core State Standards: **AP CALC AB 5.4, AP CALC AB 5.5, AP CALC AB 5.6**

Key facts

- A positive f' means f is increasing; a negative f' means f is decreasing.
- Sign changes of f' can occur only at critical numbers or where f' is undefined.
- First Derivative Test: + to – gives a relative maximum; – to + gives a relative minimum.
- A critical number without a sign change, such as $x = 0$ for x^3 , is not an extremum.

How does the derivative show increasing and decreasing?

If $f'(x) > 0$ on an interval, the graph rises as you move right, so f is **increasing**. If $f'(x) < 0$, the graph falls and f is **decreasing**. The sign of the derivative can only change at a critical number, so a sign chart organizes the work.

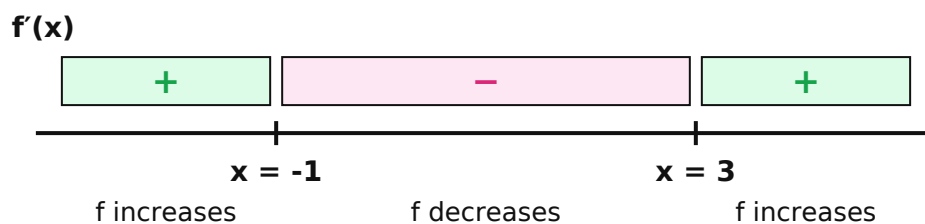


f increases where $f' > 0$ and decreases where $f' < 0$.

How do you build a sign chart?

1. Find $f'(x)$ and factor it.
2. Find the critical numbers (where $f' = 0$ or is undefined) and mark them on a number line.
3. Pick a test value in each interval and record the sign of f' .
4. Write conclusions with the intervals: increasing where $+$, decreasing where $-$.

Example. For $f(x) = \frac{x^3}{3} - x^2 - 3x + 5$, $f'(x) = x^2 - 2x - 3 = (x+1)(x-3)$. The critical numbers are -1 and 3 .

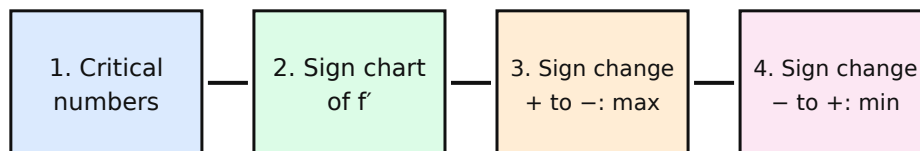


A sign chart of f' shows where f increases or decreases.

What is the First Derivative Test?

First Derivative Test

- If f' changes from **positive to negative** at c , then $f(c)$ is a **relative maximum**.
- If f' changes from **negative to positive**, then $f(c)$ is a **relative minimum**.
- If f' does not change sign, there is no extremum at c .

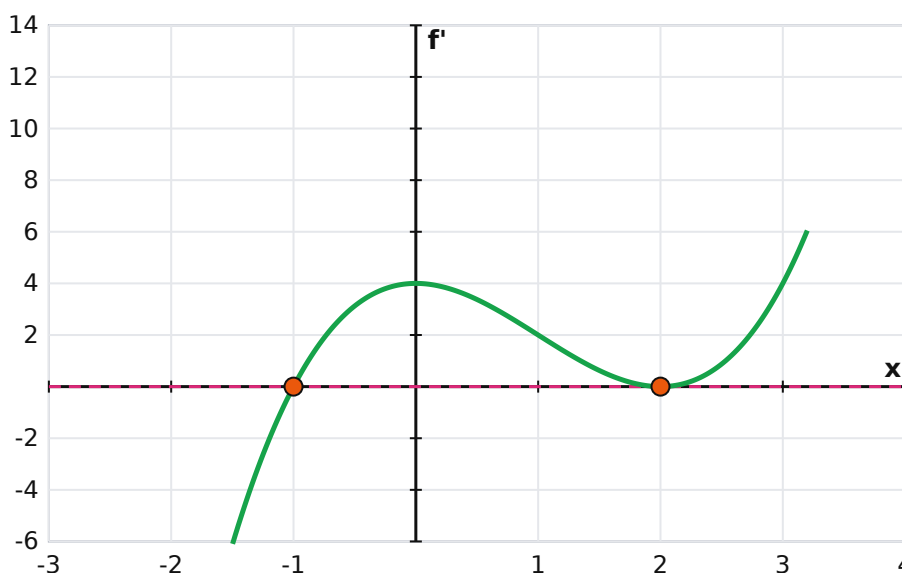


A plus-to-minus change gives a local maximum; minus-to-plus gives a local minimum.

In the example, f' goes $+, -, +$, so there is a relative maximum at $x = -1$ (value $\frac{20}{3}$) and a relative minimum at $x = 3$ (value -4).

How do you read a graph of f' ?

Look at where the graph of f' is above or below the x -axis, not at its height. In the graph below, $f'(x) = (x+1)(x-2)^2$ changes sign at $x = -1$ (minus to plus: relative minimum of f) but only touches zero at $x = 2$, with no sign change, so there is no extremum at 2.



The graph of f' : f' changes sign at -1 but only touches zero at 2.

For **absolute** extrema on a closed interval, use the candidates test from the previous lesson. On an open or infinite interval, a sign chart can show that a single relative minimum is also the absolute minimum when the function decreases before it and increases after it.

Questions about increasing, decreasing and the first derivative test

How do you find where a function is increasing?

Find the derivative, determine its sign on each interval between critical numbers, and report the intervals where the derivative is positive.

What is the First Derivative Test?

It classifies a critical number c : if f' changes from positive to negative, f has a relative maximum at c ; if it changes from negative to positive, a relative minimum; if there is no sign change, neither.

Is every critical number a maximum or minimum?

No. For $f(x) = x^3$, $f'(0) = 0$ but f' stays positive on both sides, so there is no extremum at 0.

What is the difference between relative and absolute extrema?

A relative extremum is the highest or lowest value near a point. An absolute extremum is the highest or lowest value on the entire interval being considered.

How do you read increasing and decreasing from a graph of f' ?

Check where the graph of f' lies above the x -axis (f increasing) or below it (f decreasing). The height of f' matters only for how steep f is.