

AP Calculus AB - Increasing, Decreasing and the First Derivative Test - Answer Key

Common Core AP CALC AB 5.4, AP CALC AB 5.5, AP CALC AB 5.6 · maths-pdf.fr/en/

This answer key gives a full worked solution for every problem in the first derivative test worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 5.4, AP CALC AB 5.5, AP CALC AB 5.6**

Exercise 1 - Intervals for a parabola

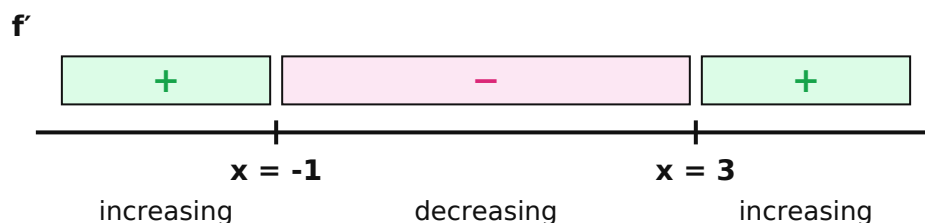
$f' = 2x - 6$, zero at $x = 3$. For $x < 3$, $f' < 0$; for $x > 3$, $f' > 0$.

Final answer: decreasing on $(-\infty, 3)$; increasing on $(3, \infty)$.

Exercise 2 - Intervals for a cubic

$f' = 3x^2 - 6x - 9 = 3(x+1)(x-3)$. Test values: $f'(-2) = 15 > 0$, $f'(0) = -9 < 0$, $f'(4) = 15 > 0$.

Final answer: increasing on $(-\infty, -1)$ and $(3, \infty)$; decreasing on $(-1, 3)$.



Exercise 3 - First derivative test, quartic

$f' = 4x^3 - 16x = 4x(x-2)(x+2)$. Critical numbers: $-2, 0, 2$.

Signs of f' : $-$ on $(-\infty, -2)$, $+$ on $(-2, 0)$, $-$ on $(0, 2)$, $+$ on $(2, \infty)$.

Final answer: relative minima $f(\pm 2) = -16$; relative maximum $f(0) = 0$.

Exercise 4 - First derivative test, exponential

$f' = e^{-x}(1-x)$, zero at $x=1$. Positive for $x < 1$, negative for $x > 1$.

Final answer: relative maximum at $x=1$, $f(1) = \frac{1}{e}$.

Exercise 5 - A function with an asymptote

$f' = \frac{2x(x-1)-x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$. Critical numbers 0 and 2; $x=1$ is not in the domain.

Signs: + on $(-\infty, 0)$, - on $(0, 1)$ and $(1, 2)$, + on $(2, \infty)$.

Final answer: relative max $f(0) = 0$; relative min $f(2) = 4$.

Exercise 6 - A square root

$f' = 1 - \frac{1}{\sqrt{x}}$, zero at $x=1$ (undefined at the endpoint 0).

Negative on $(0, 1)$, positive on $(1, \infty)$. $f(1) = 1 - 2 = -1$.

Final answer: relative (and absolute) minimum -1 at $x=1$.

Exercise 7 - A trigonometric function

$f' = 1 - 2\cos x = 0$ gives $\cos x = \frac{1}{2}$: $x = \frac{\pi}{3}$ and $\frac{5\pi}{3}$.

Signs: - on $(0, \frac{\pi}{3})$, + on $(\frac{\pi}{3}, \frac{5\pi}{3})$, - on $(\frac{5\pi}{3}, 2\pi)$.

Final answer: relative min at $x = \frac{\pi}{3}$ ($f = \frac{\pi}{3} - \sqrt{3}$); relative max at $x = \frac{5\pi}{3}$ ($f = \frac{5\pi}{3} + \sqrt{3}$).

Exercise 8 - $x \ln x$

$f' = \ln x + 1$, zero at $x = \frac{1}{e}$. Negative for $x < \frac{1}{e}$, positive after.

Final answer: relative minimum $f(\frac{1}{e}) = -\frac{1}{e}$.

Exercise 9 - Reading a graph of the derivative

$f' < 0$ for $x < -1$ and $f' > 0$ for $x > -1$ (with $f'(2) = 0$ only at a point).

At $x = -1$ the sign changes from $-$ to $+$: relative minimum. At $x = 2$ there is no sign change: no extremum.

Final answer: decreasing on $(-\infty, -1)$; increasing on $(-1, \infty)$; relative min at $x = -1$ only.

Exercise 10 - Critical but not extreme

$f' = 3x^2$ is zero at $x = 0$ and positive for every other x .

There is no sign change at 0, so f is increasing on both sides.

Final answer: critical number 0, no extremum.

Exercise 11 - Conclusions from a sign table

Increasing on $(-\infty, -2)$ and $(4, \infty)$; decreasing on $(-2, 4)$.

At -2 : $+$ to $-$, relative maximum. At 1 : $-$ to $-$, no extremum. At 4 : $-$ to $+$, relative minimum.

Final answer: rel. max at -2 ; no extremum at 1 ; rel. min at 4 .

Exercise 12 - Relative versus absolute

$f' = 6(x-1)(x-2)$: $+, -, +$ around 1 and 2. Relative maximum $f(1) = 5$, relative minimum $f(2) = 4$.

Candidates for absolute extrema: $f(0) = 0$, $f(1) = 5$, $f(2) = 4$, $f(3) = 9$.

Final answer: absolute max 9 at $x = 3$; absolute min 0 at $x = 0$.

Exercise 13 - Minimum on an open interval

$f' = 1 - \frac{1}{x^2} = \frac{(x-1)(x+1)}{x^2}$. On $(0, \infty)$, the only critical number is $x = 1$.

$f' < 0$ on $(0, 1)$ and $f' > 0$ on $(1, \infty)$, so f decreases then increases.

Final answer: absolute minimum $f(1)=2$.

Exercise 14 - Finding coefficients

$f' = 3x^2 + 2ax + b$ must equal $3(x-1)(x-3) = 3x^2 - 12x + 9$.

Compare: $2a = -12$ and $b = 9$.

Final answer: $a = -6$, $b = 9$.

Exercise 15 - Ball height

$s' = -32t + 64 = -32(t-2)$. Positive for $t < 2$, negative for $t > 2$.

Final answer: rising on $(0,2)$, falling on $(2,4)$; max height $s(2) = 64$ ft.

Exercise 16 - Average cost

$A' = 1 - \frac{100}{x^2} = 0$ gives $x = 10$. Negative on $(0,10)$, positive on $(10,\infty)$.

$A(10) = 10 + 10 = 20$.

Final answer: 10 hoodies, minimum average cost \$20.

Exercise 17 - Hiker elevation

E' changes from $+$ to $-$ at $t = 2$ and from $-$ to $+$ at $t = 5$.

Final answer: local high point at $t = 2$ h; local low point at $t = 5$ h.

Exercise 18 - Profit on an interval

$P' = -3x^2 + 18x + 48 = -3(x-8)(x+2)$. The only critical number in the interval is $x = 8$.

Candidates: $P(0) = 0$, $P(8) = 448$, $P(10) = 380$.

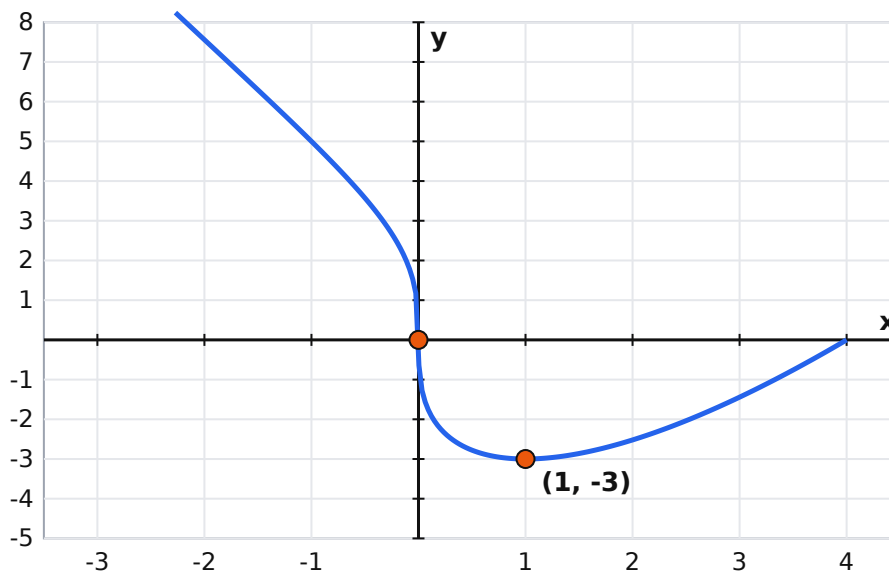
Final answer: 8 backpacks, profit \$448.

Exercise 19 - Critical number with undefined derivative

$f' = \frac{4}{3}x^{1/3} - \frac{4}{3}x^{-2/3} = \frac{4(x-1)}{3x^{2/3}}$. Critical numbers: $x=1$ (zero) and $x=0$ (undefined, but in the domain).

Since $x^{2/3} > 0$ for $x \neq 0$, the sign of f' is the sign of $x-1$: negative on $(-\infty, 0)$ and $(0, 1)$, positive on $(1, \infty)$.

Final answer: relative min $f(1) = -3$ at $x=1$; no extremum at $x=0$.



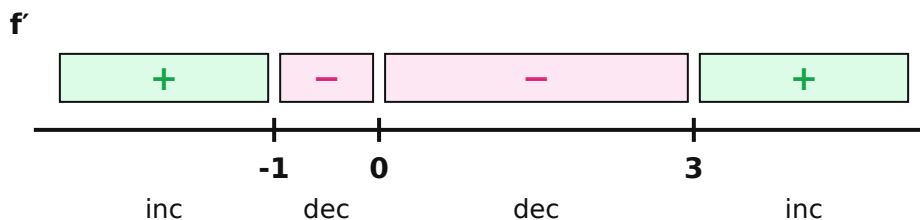
Vertical tangent at 0, minimum at 1.

Exercise 20 - Repeated factor

Critical numbers: $-1, 0, 3$. Signs: $+$ on $(-\infty, -1)$, $-$ on $(-1, 0)$, $-$ on $(0, 3)$, $+$ on $(3, \infty)$.

The factor x^2 never changes sign, so $x=0$ is not an extremum.

Final answer: increasing on $(-\infty, -1)$ and $(3, \infty)$; rel. max at -1 ; rel. min at 3 ; none at 0 .



No sign change at $x = 0$.