

AP Calculus AB - Increasing, Decreasing and the First Derivative Test - Worksheet

Common Core AP CALC AB 5.4, AP CALC AB 5.5, AP CALC AB 5.6 · maths-pdf.fr/en/

Name: _____ Class: _____ Date: ____/____/____

These 20 problems practice increasing and decreasing intervals, sign charts of the derivative, the First Derivative Test and absolute extrema (AP Calculus AB, topics 5.4, 5.5, 5.6). The early problems build the basic skill, and each one uses a different function or situation so that no two feel alike.

Later problems read the graph of the derivative, use repeated factors with no sign change, and apply the test to situations such as average cost and a hiker. Work in pencil, show every step, and check the answer key only after a full attempt.

AP Calculus AB · Common Core State Standards: **AP CALC AB 5.4, AP CALC AB 5.5, AP CALC AB 5.6**

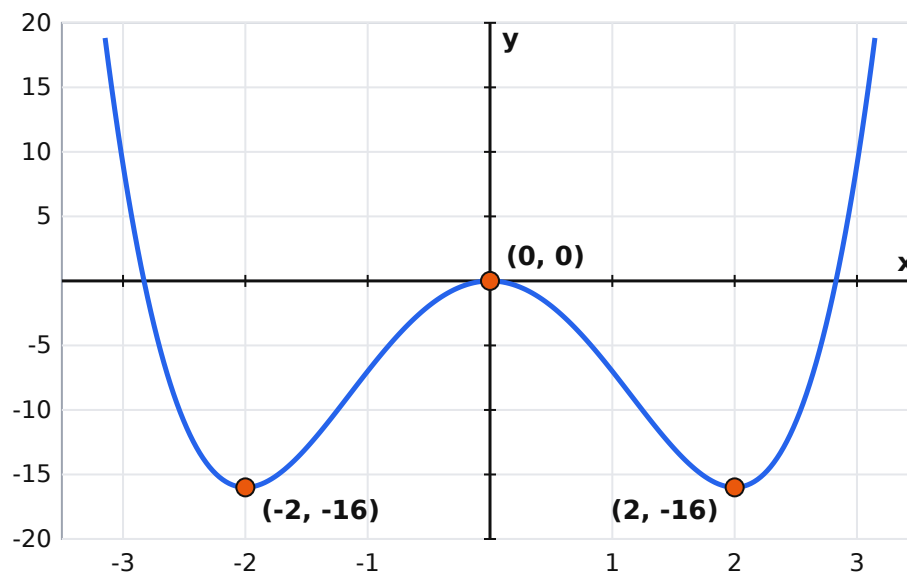
Exercise 1 - Intervals for a parabola

Find the intervals where $f(x) = x^2 - 6x + 5$ is increasing and decreasing.

Exercise 2 - Intervals for a cubic

Find where $f(x) = x^3 - 3x^2 - 9x + 5$ is increasing and decreasing.

Exercise 3 - First derivative test, quartic



The graph of $f(x) = x^4 - 8x^2$.

Use the First Derivative Test to classify the critical points of $f(x) = x^4 - 8x^2$.

Exercise 4 - First derivative test, exponential

Find the relative extremum of $f(x) = xe^{-x}$.

Exercise 5 - A function with an asymptote

Find the intervals of increase and decrease and the relative extrema of $f(x) = \frac{x^2}{x-1}$.

Exercise 6 - A square root

Find the relative extremum of $f(x) = x - 2\sqrt{x}$ for $x \geq 0$.

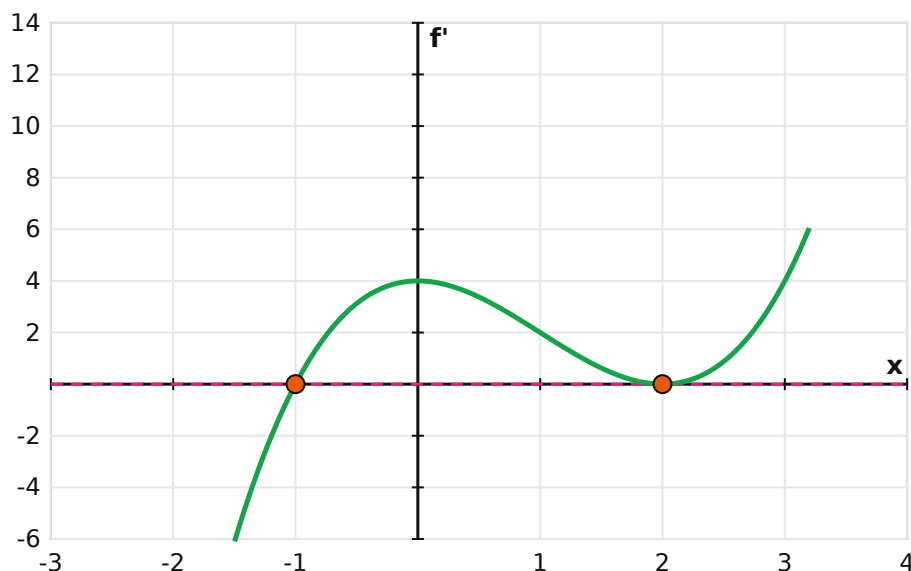
Exercise 7 - A trigonometric function

Find the intervals of increase and decrease and the relative extrema of $f(x) = x - 2\sin x$ on $(0, 2\pi)$.

Exercise 8 - $x \ln x$

Find the relative extremum of $f(x) = x \ln x$ for $x > 0$.

Exercise 9 - Reading a graph of the derivative



The graph of f' .

The graph shows $f'(x) = (x+1)(x-2)^2$. Find the intervals where f is increasing and decreasing, and classify the critical points.

Exercise 10 - Critical but not extreme

Show that $f(x) = x^3$ has a critical number but no relative extremum.

Exercise 11 - Conclusions from a sign table

interval	$(-\infty, -2)$	$(-2, 1)$	$(1, 4)$	$(4, \infty)$
sign of f'	+	−	−	+

Signs of f' on each interval ($f' = 0$ at $-2, 1$ and 4).

The table shows the sign of f' , and f is continuous. Describe where f increases or decreases and classify each critical number $-2, 1, 4$.

Exercise 12 - Relative versus absolute

Find the relative and absolute extrema of $f(x) = 2x^3 - 9x^2 + 12x$ on $[0, 3]$.

Exercise 13 - Minimum on an open interval

Show that $f(x) = x + \frac{1}{x}$ has an absolute minimum on $(0, \infty)$ and find it.

Exercise 14 - Finding coefficients

The function $f(x) = x^3 + ax^2 + bx$ has critical numbers $x = 1$ and $x = 3$. Find a and b .

Exercise 15 - Ball height



Height of a ball in feet.

Liam throws a ball so that its height in feet after t seconds is $s(t) = -16t^2 + 64t$. When is the ball rising? Falling? What is its maximum height?

Exercise 16 - Average cost

The average cost in dollars per hoodie for an order of x hoodies is $A(x) = x + \frac{100}{x}$ for $x > 0$. Find the order size that minimizes the average cost, and justify that it is a minimum.

Exercise 17 - Hiker elevation

interval (hours)	(0, 2)	(2, 5)	(5, 8)
sign of E'	+	-	+

Sign of the elevation rate $E'(t)$.

A hiker's elevation is $E(t)$ feet after t hours, $0 \leq t \leq 8$. The table gives the sign of $E'(t)$. When is the hiker at a local high point or a local low point?

Exercise 18 - Profit on an interval

The weekly profit in dollars from making x handmade backpacks is $P(x) = -x^3 + 9x^2 + 48x$ for $0 \leq x \leq 10$. Find the number of backpacks that maximizes the profit.

Exercise 19 - Critical number with undefined derivative

Let $f(x) = x^{4/3} - 4x^{1/3}$. Find all critical numbers and classify them.

Exercise 20 - Repeated factor

The derivative of f is $f'(x) = x^2(x-3)(x+1)$. Find the intervals where f is increasing and classify each critical number.