

AP Calculus AB - Increasing, Decreasing and the First Derivative Test - Test Answer Key

Common Core AP CALC AB 5.4, AP CALC AB 5.5, AP CALC AB 5.6 · maths-pdf.fr/en/

Use this answer key to score the first derivative test test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

AP Calculus AB · Common Core State Standards: **AP CALC AB 5.4, AP CALC AB 5.5, AP CALC AB 5.6**

Question 1 - Intervals of monotonicity [2 pts]

$f' = e^x + xe^x = e^x(x+1)$. Since $e^x > 0$, the sign of f' is the sign of $x+1$.

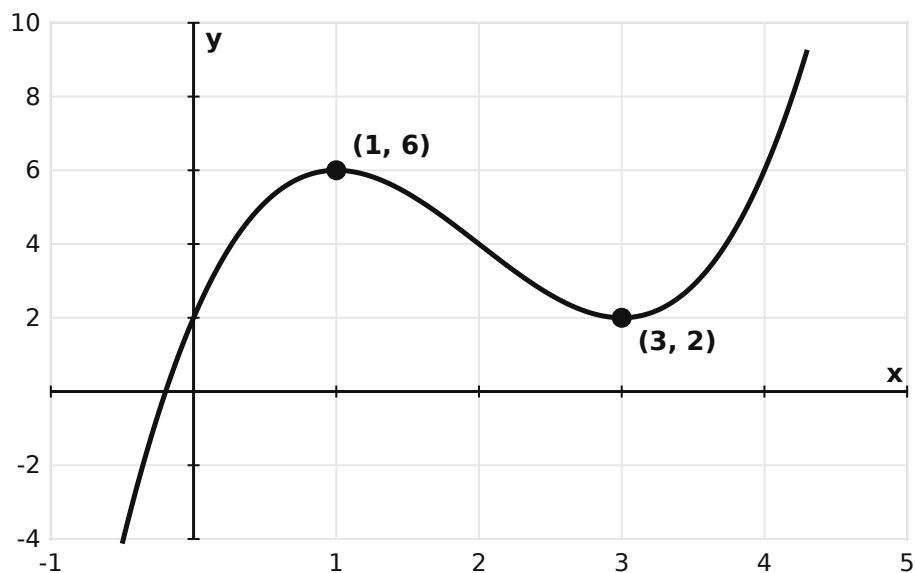
Final answer: **decreasing on $(-\infty, -1)$; increasing on $(-1, \infty)$.**

Scoring: 1 point for the derivative and critical number; 1 point for the intervals.

Question 2 - First derivative test [3 pts]

$f' = 3x^2 - 12x + 9 = 3(x-1)(x-3)$. Signs: + on $(-\infty, 1)$, - on $(1, 3)$, + on $(3, \infty)$.

Final answer: **relative maximum $f(1) = 6$; relative minimum $f(3) = 2$.**



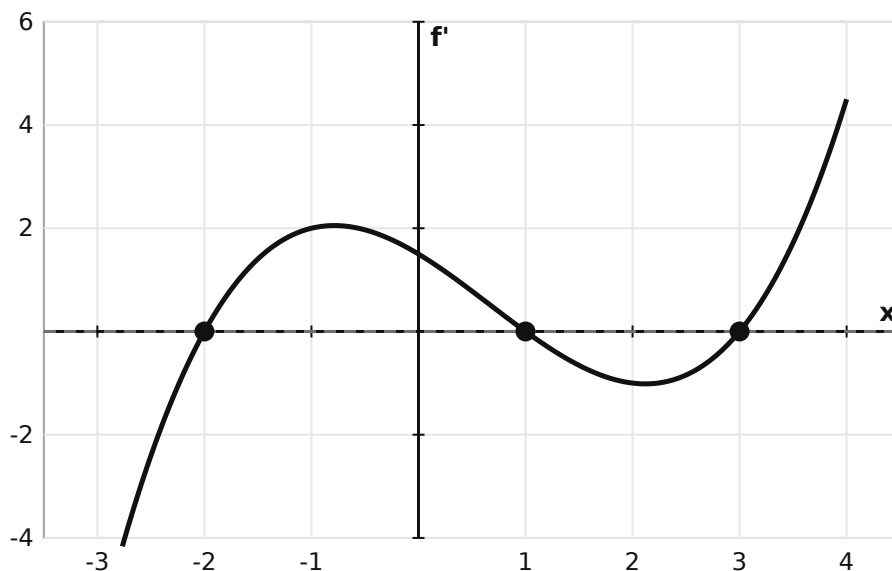
Graph of $f(x) = x^3 - 6x^2 + 9x + 2$.

Scoring: 1 point for f and the critical numbers; 1 point for the sign chart; 1 point for the classification with values.

Question 3 - Reading the derivative graph [2 pts]

For $x < -2$, $f' < 0$; on $(-2, 1)$, $f' > 0$; on $(1, 3)$, $f' < 0$; for $x > 3$, $f' > 0$.

- $(-2, 1)$ and $(3, \infty)$.
- $x = 1$, where f' changes from positive to negative.



The graph of f' (not f).

Scoring: 1 point per part.

Question 4 - No sign change [3 pts]

- a. $x=1$ and $x=-4$.
- b. $g'<0$ on $(-\infty,-4)$, $g'>0$ on $(-4,1)$ and $(1,\infty)$.
- c. At -4 : $-$ to $+$, **relative minimum**. At 1 : no sign change, **no extremum**.

Scoring: 1 point per part.

Question 5 - Absolute extrema [3 pts]

$f' = 4x^3 - 12x^2 = 4x^2(x-3)$. Critical numbers: 0 and 3.

Candidates: $f(-1)=5$, $f(0)=0$, $f(3)=-27$, $f(4)=0$.

Final answer: **absolute maximum 5 at $x = -1$; absolute minimum -27 at $x = 3$.**

Scoring: 1 point for the critical numbers; 1 point for the four values; 1 point for the conclusions.

Question 6 - Justification [2 pts]

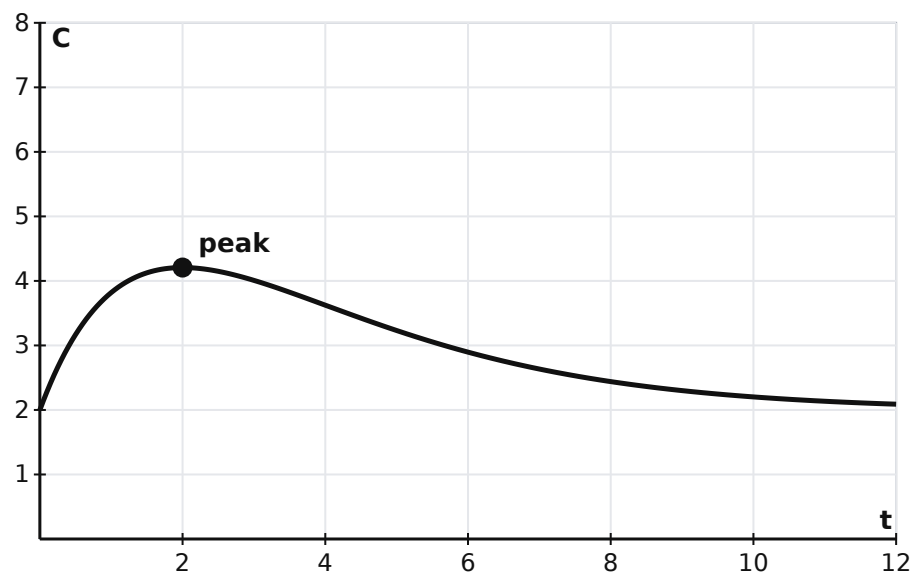
The denominator is always positive, so the sign of h' is the sign of $x-2$: negative for $x<2$ and positive for $x>2$.

Since h' changes from negative to positive at $x=2$, **h has a relative minimum at $x = 2$.**

Scoring: 1 point for the sign analysis; 1 point for the conclusion with justification.

Question 7 - Concentration over time [3 pts]

- a. $C'(t) = 3e^{-t/2} - \frac{3}{2}te^{-t/2} = \frac{3}{2}e^{-t/2}(2-t)$.
- b. Increasing on $(0,2)$ (positive), decreasing on $(2,\infty)$.
- c. $C(2) = 2 + 6e^{-1} \approx 4.21$ mg/L.



Concentration $C(t)$ over time.

Scoring: 1 point per part.

Question 8 - Finding constants [2 pts]

$f' = 3x^2 + 2px + q$ has zeros at -1 and 3 , so $f' = 3(x+1)(x-3) = 3x^2 - 6x - 9$.

Therefore $p = -3$ **and** $q = -9$.

Scoring: 1 point for the derivative with both zeros; 1 point for p and q .

Total

20 points