

AP Calculus AB - Fundamental Theorem of Calculus and Antiderivatives - Lesson

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The Fundamental Theorem of Calculus connects derivatives and integrals. Part 1 says the derivative of the accumulation function from a to x equals $f(x)$. Part 2 says a definite integral equals $F(b) - F(a)$, where F is any antiderivative of f . An indefinite integral adds the constant C , found from an initial condition.

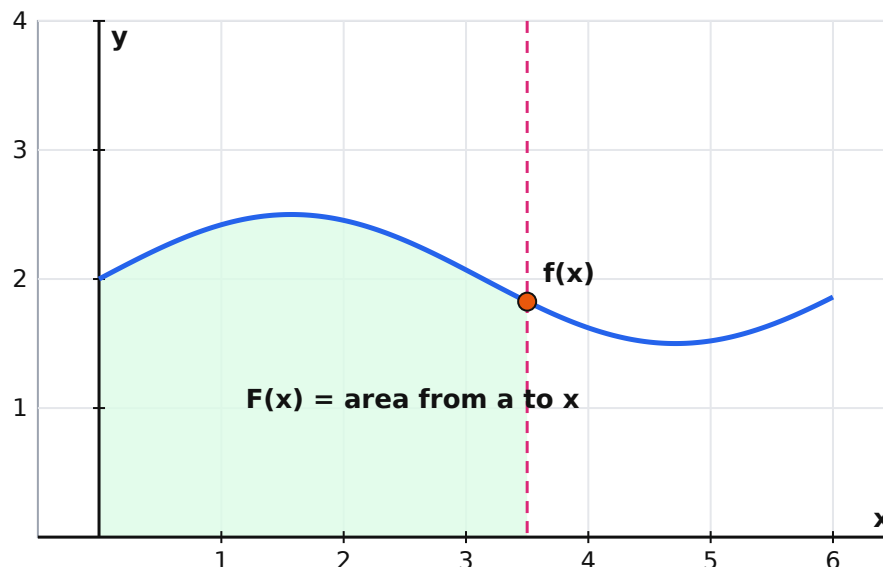
AP Calculus AB · Common Core State Standards: AP CALC AB 6.4, AP CALC AB 6.5, AP CALC AB 6.6, AP CALC AB 6.7, AP CALC AB 6.8

Key facts

- FTC Part 1: d/dx of the integral from a to x of $f(t) dt$ equals $f(x)$.
- FTC Part 2: the integral from a to b of f equals $F(b) - F(a)$ for any antiderivative F .
- For an upper limit $g(x)$, multiply by $g'(x)$ (chain rule).
- Every indefinite integral needs $+ C$; an initial condition determines C .

What is an accumulation function?

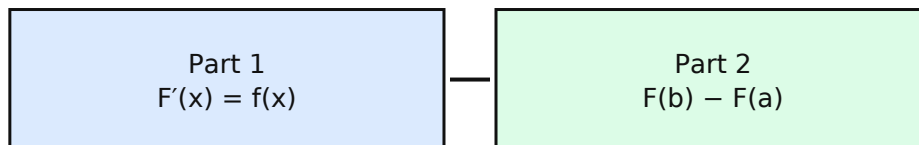
Define $F(x) = \int_a^x f(t) dt$. It records the area (signed) accumulated between a and x . As x moves right, the area grows at a rate equal to the height of the graph.



$F(x)$ is the area accumulated up to x ; its rate of growth is the height $f(x)$.

Fundamental Theorem of Calculus

- **Part 1:** If f is continuous, $\frac{d}{dx} \int_a^x f(t) dt = f(x)$.
- **Part 2:** If $F' = f$, then $\int_a^b f(x) dx = F(b) - F(a)$.



The two parts of the Fundamental Theorem link derivatives and integrals.

When the upper limit is a function $g(x)$, use the chain rule: $\frac{d}{dx} \int_a^{g(x)} f(t) dt = f(g(x)) \cdot g'(x)$. For example, $\frac{d}{dx} \int_0^{x^2} \cos t dt = 2x \cos(x^2)$.

What is an antiderivative?

An antiderivative of f is a function F with $F' = f$. The family of all antiderivatives is the **indefinite integral** $\int f(x) dx = F(x) + C$. The constant C appears because constants have derivative zero.

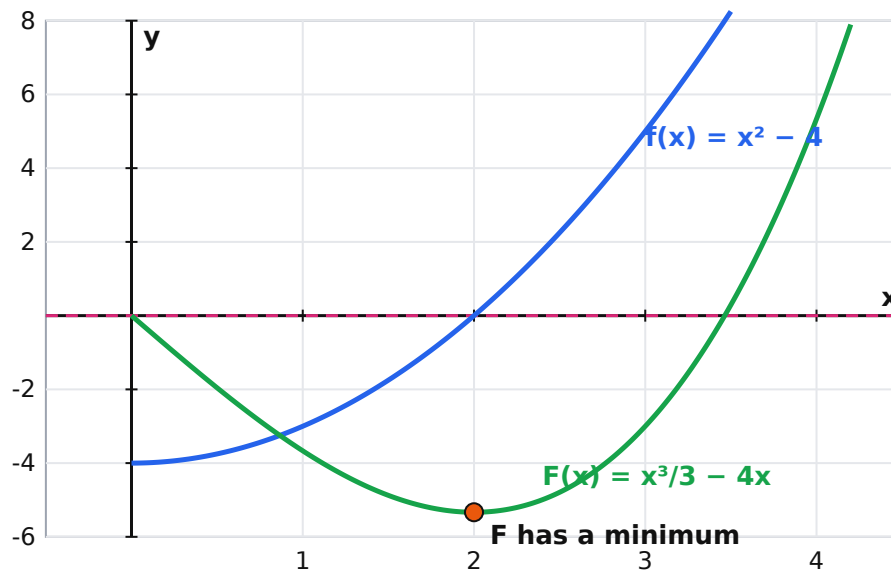
$f(x)$	antiderivative $F(x)$
x^n ($n \neq -1$)	$x^{n+1} / (n + 1)$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
e^x	e^x
$1/x$	$\ln x $
$\sec^2 x$	$\tan x$
$1/(1 + x^2)$	$\tan^{-1} x$

Basic antiderivatives to memorize (add $+ C$ for indefinite integrals).

How do you evaluate a definite integral?

1. Find an antiderivative F (no constant needed).
2. Evaluate $F(b) - F(a)$.
3. Interpret the sign and units.

Example. $\int_1^3 (2x+1) dx = [x^2+x]_1^3 = (9+3)-(1+1) = 10$. Notice that the graph of $F(x) = \frac{x^3}{3} - 4x$ (green) falls while $f(x) = x^2 - 4$ (blue) is negative.



F decreases while f is negative and increases when f is positive; F has its minimum where f changes sign.

The theorem also explains *net change*: if $r(t)$ is a rate, then $\int_a^b r(t) dt$ is the total change in the quantity from time a to time b . A tank with inflow rate $12-2t$ gallons per minute gains $\int_0^6 (12-2t) dt = 36$ gallons in 6 minutes, and the answer carries the units gallons because (gallons per minute) times (minutes) is gallons.

- Do not forget the constant of integration for an indefinite integral.
- For a definite integral, write the antiderivative in brackets and subtract the lower value from the upper value.
- If the graph crosses the axis, the integral is net (signed) area; split the interval to find total area.

What is an initial value problem?

If you know $f'(x)$ and a single value of f , integrate to get $f(x)+C$ and use the value to find C . For $f'(x) = 6x^2 - 2$ and $f(1) = 5$: $f(x) = 2x^3 - 2x + C$, and $0 + C = 5$ gives $C = 5$.

Questions about fundamental theorem of calculus and antiderivatives

What does the Fundamental Theorem of Calculus say?

It links differentiation and integration: the derivative of the accumulation function equals the integrand, and a definite integral equals the change in any antiderivative between the limits.

How do you find a derivative of an integral with a variable upper limit?

Substitute the upper limit into the integrand and multiply by the derivative of the upper limit. For the integral from 0 to x^2 of $\cos t \, dt$, the derivative is $\cos(x^2)$ times $2x$.

Why do indefinite integrals include + C?

Functions that differ by a constant have the same derivative, so an antiderivative is determined only up to a constant. The + C represents that family.

What is the antiderivative of $1/x$?

It is the natural logarithm of the absolute value of x , $\ln|x| + C$. It is the missing case in the power rule for integrals.

How do you solve an initial value problem?

Find the general antiderivative with + C, substitute the given point to solve for C, and then write the particular function.