

AP Calculus AB - Fundamental Theorem of Calculus and Antiderivatives - Answer Key

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This answer key gives a full worked solution for every problem in the fundamental theorem worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 6.4, AP CALC AB 6.5, AP CALC AB 6.6, AP CALC AB 6.7, AP CALC AB 6.8**

Exercise 1 - Polynomial antiderivative

Use the power rule term by term: $\int x^n dx = \frac{x^{n+1}}{n+1}$.

Check by differentiating $x^3 - 2x^2 + 5x$: you get $3x^2 - 4x + 5$.

Final answer: $x^3 - 2x^2 + 5x + C$.

Exercise 2 - Negative powers

Rewrite $\frac{2}{x^2} = 2x^{-2}$. Then $\int 2x^{-2} dx = \frac{2x^{-1}}{-1} = -\frac{2}{x}$.

Final answer: $\frac{x^4}{4} - \frac{2}{x} + C$.

Exercise 3 - Roots

Rewrite as $x^{1/2} + x^{-1/2}$. Then $\frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2}$.

Final answer: $\frac{2}{3}x^{3/2} + 2\sqrt{x} + C$.

Exercise 4 - Sine and cosine

$\int \sin x \, dx = -\cos x$ and $\int \cos x \, dx = \sin x$.

Final answer: $-\cos x - 2\sin x + C$.

Exercise 5 - Exponential and reciprocal

$$\int e^x dx = e^x \text{ and } \int \frac{1}{x} dx = \ln|x|.$$

Final answer: $e^x + 3\ln|x| + C$.

Exercise 6 - Trig and inverse trig forms

a. $\tan x + C$ since $(\tan x)' = \sec^2 x$.

b. $\tan^{-1} x + C$ since $(\tan^{-1} x)' = \frac{1}{1+x^2}$.

Exercise 7 - Definite integral, linear

$$F(x) = x^2 + x. \quad F(3) - F(1) = 12 - 2 = 10.$$

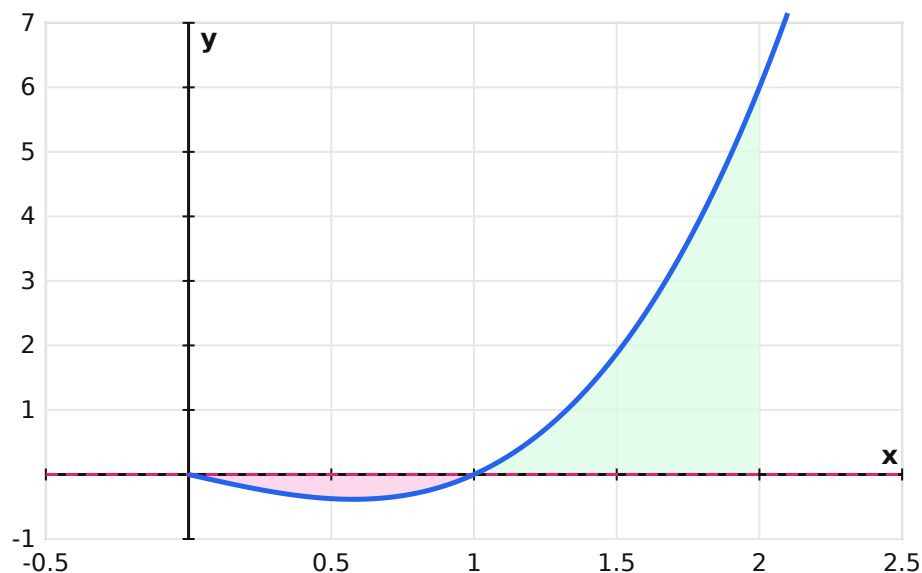
Geometry: a trapezoid with parallel sides 3 and 7 and width 2 has area $\frac{3+7}{2} \cdot 2 = 10$.

Final answer: 10.

Exercise 8 - Definite integral, cubic

$$F = \frac{x^4}{4} - \frac{x^2}{2}. \quad F(2) = 4 - 2 = 2 \text{ and } F(0) = 0.$$

Final answer: 2.



Signed areas: $-1/4$ and $9/4$ give 2.

Exercise 9 - Definite integral, root

$$F = \frac{2}{3}x^{3/2}. \quad F(4) = \frac{2}{3}(8) = \frac{16}{3} \quad \text{and} \quad F(1) = \frac{2}{3}.$$

Final answer: $\frac{14}{3}$.

Exercise 10 - Definite integral, sine

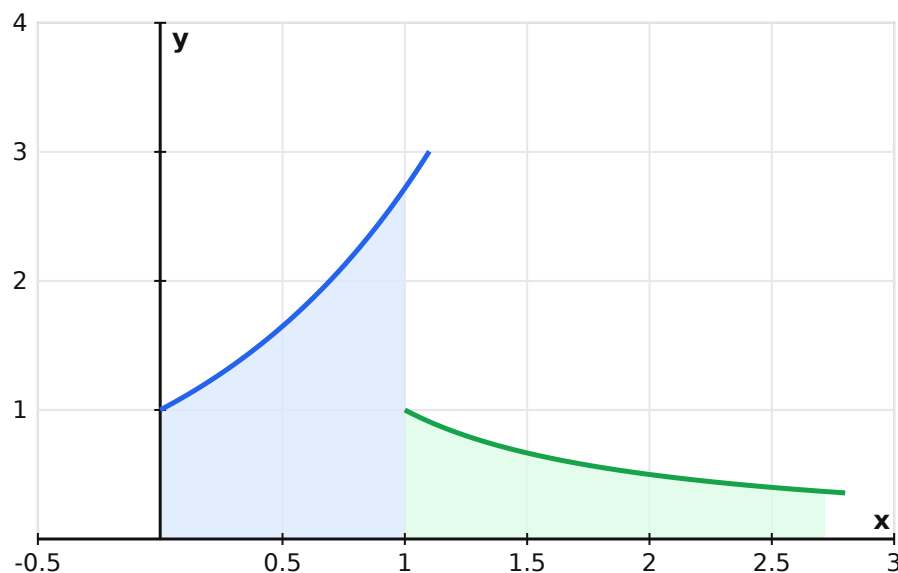
$$[-\cos x]_0^\pi = -\cos \pi - (-\cos 0) = 1 + 1 = 2.$$

Final answer: 2.

Exercise 11 - Exponential and logarithm integrals

a. $[e^x]_0^1 = e - 1 \approx 1.718.$

b. $[\ln x]_1^e = 1 - 0 = 1.$



Two regions: area $e - 1$ and area 1.

Exercise 12 - Absolute value

Split at 0: $\int_{-1}^0 (-x) dx = \frac{1}{2}$ and $\int_0^2 x dx = 2$.

Final answer: $\frac{1}{2} + 2 = \frac{5}{2}$.

Exercise 13 - FTC part 1, direct

By FTC Part 1, $F'(x) = \sqrt{x^2 + 3}$.

$$F'(1) = \sqrt{4} = 2.$$

Final answer: $F'(x) = \sqrt{x^2 + 3}$, $F'(1) = 2$.

Exercise 14 - FTC part 1, chain rule

Substitute the upper limit and multiply by its derivative: $\cos(x^2) \cdot 2x$.

Final answer: $2x \cos(x^2)$.

Exercise 15 - FTC part 1, variable lower limit

Swap the limits to put the variable on top: $\int_x^5 = -\int_5^x$.

So the derivative is $-(x^3+1)$.

Final answer: $-(x^3+1)$.

Exercise 16 - Accumulation from a graph

$g(2) = 2 \cdot 2 = 4$. From 2 to 3 the average height is $\frac{2+0}{2} = 1$, so $g(3) = 5$. From 3 to 4 the average height is -1 : $g(4) = 4$. From 4 to 6: $-2 \cdot 2 = -4$, so $g(6) = 0$.

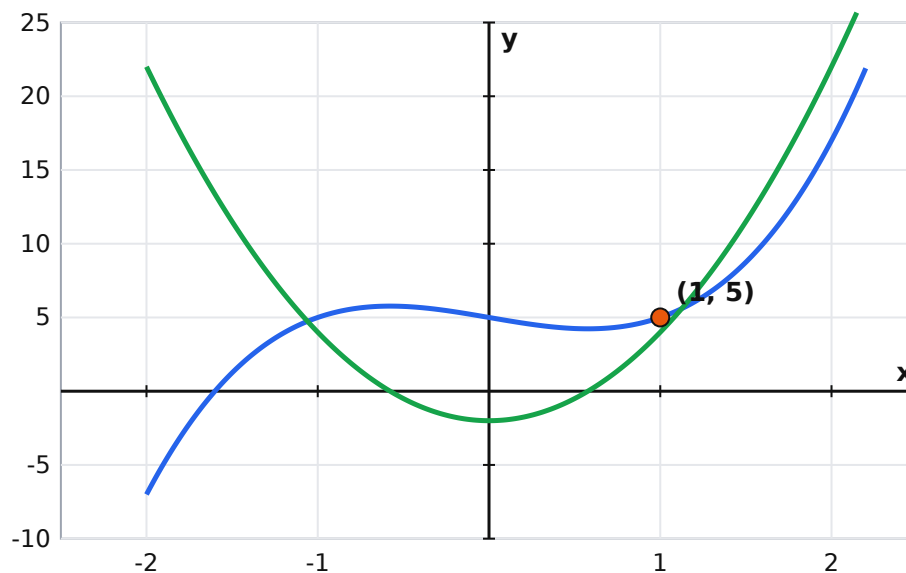
$g' = f$ changes from positive to negative at $t = 3$, so g is largest at $x = 3$.

Final answer: $g(2) = 4$, $g(3) = 5$, $g(4) = 4$, $g(6) = 0$; maximum at $x = 3$.

Exercise 17 - Initial value problem

$f(x) = 2x^3 - 2x + C$. $f(1) = 0 + C = 5$, so $C = 5$.

Final answer: $f(x) = 2x^3 - 2x + 5$.



The particular solution passes through (1, 5).

Exercise 18 - Second-order initial value problem

$f' = 6x^2 - 6x + C_1$ and $f'(0) = 1$ gives $C_1 = 1$.

$f = 2x^3 - 3x^2 + x + C_2$ and $f(0) = 2$ gives $C_2 = 2$.

Final answer: $f(x) = 2x^3 - 3x^2 + x + 2$.

Exercise 19 - Position from velocity

$s(t) = t^3 - 6t^2 + 9t + C$ with $s(0) = C = 2$.

$s(4) = 64 - 96 + 36 + 2 = 6$.

Final answer: $s(t) = t^3 - 6t^2 + 9t + 2$; $s(4) = 6$ ft.

Exercise 20 - Net change of water

Net change: $\int_0^6 (12 - 2t) dt = [12t - t^2]_0^6 = 72 - 36 = 36$.

Check with geometry: a triangle with base 6 and height 12 has area 36.

Final answer: $50 + 36 = 86$ gallons.

Exercise 21 - Accumulation function analysis

$F'(x) = x^2 - 4 = (x-2)(x+2)$. Signs: + for $x < -2$, - for $-2 < x < 2$, + for $x > 2$.

$F(x) = \frac{x^3}{3} - 4x$, so $F(-2) = \frac{16}{3}$ and $F(2) = -\frac{16}{3}$.

Final answer: increasing on $(-\infty, -2), (2, \infty)$; relative max at $x = -2$; relative min at $x = 2$.

Exercise 22 - Area with a sign change

$t^2 - 4$ changes sign at $t = 2$. On $[0, 2]$: $\int_0^2 (4 - t^2) dt = 8 - \frac{8}{3} = \frac{16}{3}$.

On $[2, 3]$: $\int_2^3 (t^2 - 4) dt = (9 - 12) - \left(\frac{8}{3} - 8\right) = \frac{7}{3}$.

Final answer: $\frac{16}{3} + \frac{7}{3} = \frac{23}{3}$.

Exercise 23 - Total cost from marginal cost

$\int_{50}^{100} (40 - 0.2x) dx = [40x - 0.1x^2]_{50}^{100} = (4000 - 1000) - (2000 - 250) = 3000 - 1750 = 1250$.

Final answer: \$1250.