

AP Calculus AB - Fundamental Theorem of Calculus and Antiderivatives - Test Answer Key

Common Core AP CALC AB 6.4, AP CALC AB 6.5, AP CALC AB 6.6, AP CALC AB 6.7, AP CALC AB 6.8 · maths-pdf.fr/en/

Use this answer key to score the fundamental theorem test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

AP Calculus AB · Common Core State Standards: **AP CALC AB 6.4, AP CALC AB 6.5, AP CALC AB 6.6, AP CALC AB 6.7, AP CALC AB 6.8**

Question 1 - Indefinite integral [2 pts]

$$2x^4 - 3x^2 + \ln|x| + C.$$

Scoring: 1 point for the polynomial terms; 1 point for $\ln|x|$ and $+ C$.

Question 2 - Definite integral [2 pts]

$$[\sin x + 2x]_0^{\pi/2} = (1 + \pi) - 0 = 1 + \pi.$$

Final answer: $1 + \pi$.

Scoring: 1 point for the antiderivative; 1 point for the value.

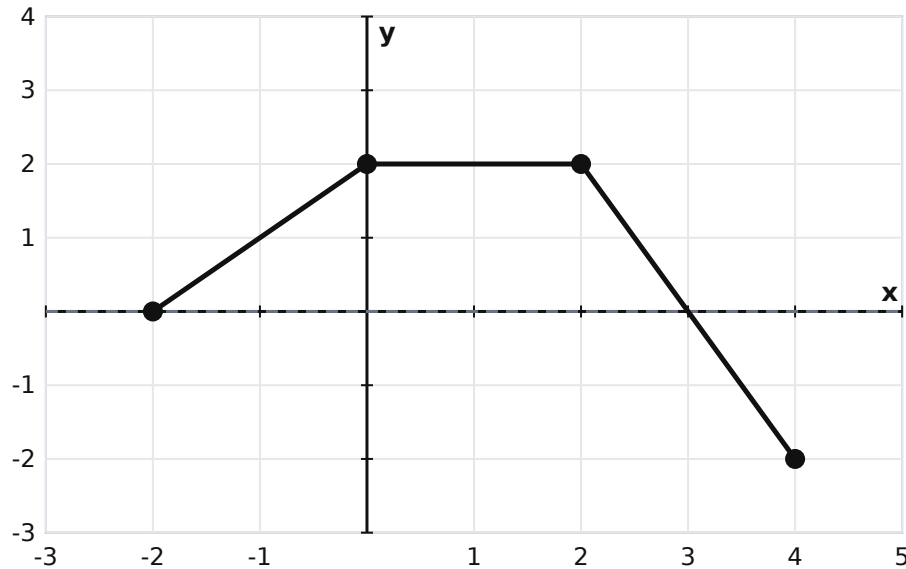
Question 3 - FTC with the chain rule [3 pts]

- a. $G'(x) = \sqrt{1+(x^3)^2} \cdot 3x^2 = 3x^2\sqrt{1+x^6}.$
- b. $G'(1) = 3\sqrt{2}.$

Scoring: (a) 1 point for the integrand at the upper limit; 1 point for the chain factor $3x^2$; (b) 1 point.

Question 4 - Accumulation from a graph [3 pts]

- a. $g(2) = 2 \cdot 2 = 4$. $g(-2) = -\int_{-2}^0 f = -\frac{1}{2}(2)(2) = -2$.
- b. $g' = f$ changes from positive to negative at $x = 3$ (where $f = 0$ on the falling segment: $2 - 2(x - 2) = 0$ gives $x = 3$). So $x = 3$.



The graph of f .

Scoring: (a) 1 point per value; (b) 1 point for the sign change of f .

Question 5 - Initial value problem [2 pts]

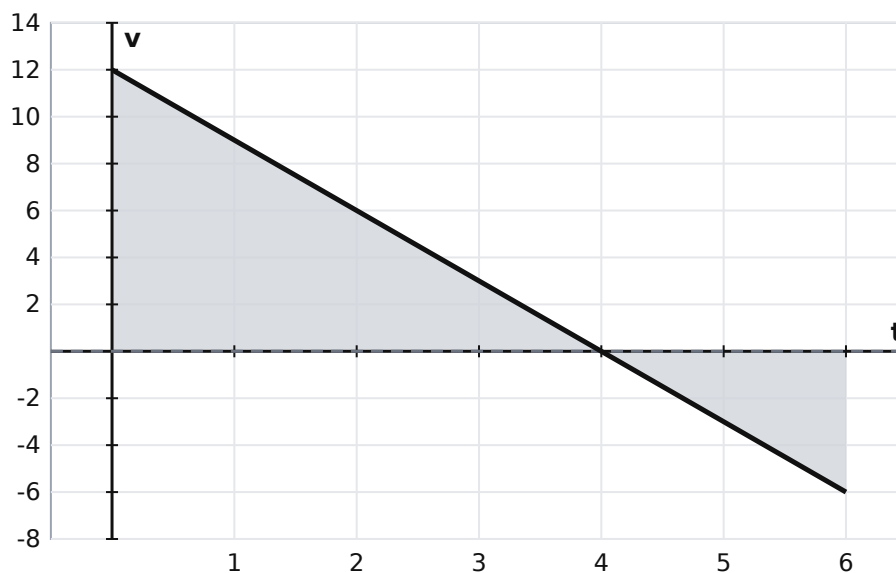
$$f = x^4 - 3x + C. \quad f(2) = 16 - 6 + C = 10, \text{ so } C = 0.$$

$$\text{Final answer: } f(x) = x^4 - 3x.$$

Scoring: 1 point for the antiderivative; 1 point for C .

Question 6 - Motion with integrals [3 pts]

- a. $s(t) = 12t - \frac{3}{2}t^2 + 5$.
- b. $\int_0^6 (12 - 3t) dt = 72 - 54 = 18$ ft.
- c. $v = 0$ at $t = 4$. $\int_0^4 = 48 - 24 = 24$ and $\left| \int_4^6 \right| = |(72 - 54) - (48 - 24)| = 6$, so the distance is 30 ft.



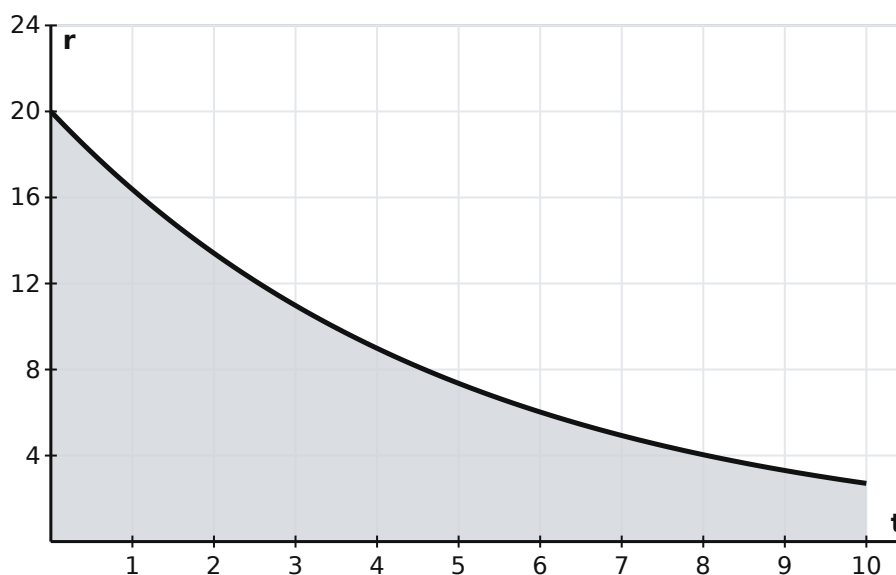
Velocity in feet per second.

Scoring: 1 point per part.

Question 7 - Net change in context [3 pts]

a. $\int_0^{10} 20e^{-0.2t} dt$.

b. $[-100e^{-0.2t}]_0^{10} = -100e^{-2} + 100 = 100(1 - e^{-2}) \approx 86.47$ gallons.



Leak rate in gallons per hour.

Scoring: 1 point for the integral; 1 point for the antiderivative; 1 point for the value.

Question 8 - Properties [2 pts]
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$$F(6) - F(2) = 9 \text{ gives } F(6) = 13. \int_6^2 3f = -3(9) = -27.$$

Scoring: 1 point for $F(6)$; 1 point for the integral.

Total
20 points