

AP Calculus AB - Implicit Differentiation - Lesson

Common Core AP CALC AB 3.2 · maths-pdf.fr/en/

Implicit differentiation finds dy/dx for an equation relating x and y without solving for y . Differentiate both sides with respect to x , multiply by dy/dx every time you differentiate a y term (chain rule), then solve for dy/dx . For $x^2 + y^2 = 25$, the result is $dy/dx = -x/y$.

AP Calculus AB · Common Core State Standards: **AP CALC AB 3.2**

Key facts

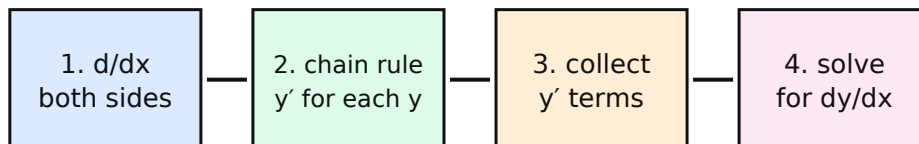
- Differentiate every term with respect to x ; a y term gets a factor dy/dx .
- Use the product rule for terms such as xy , and the chain rule for $\sin y$ or e^y .
- The derivative may contain both x and y ; substitute a point on the curve to get a slope.
- Horizontal tangents: numerator zero. Vertical tangents: denominator zero.

What is implicit differentiation?

Some curves, such as the circle $x^2 + y^2 = 25$, cannot be written as a single function $y = f(x)$. The equation defines y *implicitly*. Implicit differentiation finds $\frac{dy}{dx}$ without solving for y first.

Key idea

Treat y as a function of x . Whenever you differentiate a term with y , use the chain rule: $\frac{d}{dx}[y^n] = ny^{n-1}\frac{dy}{dx}$. Terms with only x are differentiated as usual.



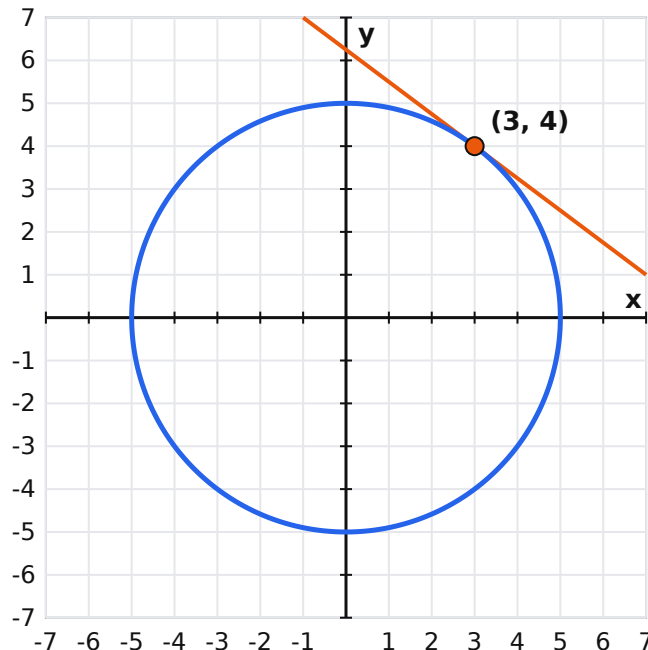
Four steps for implicit differentiation.

How do you do it step by step?

Example. Find $\frac{dy}{dx}$ for $x^2 + y^2 = 25$.

1. Differentiate both sides: $2x + 2y \frac{dy}{dx} = 0$.
2. Isolate the $\frac{dy}{dx}$ term: $2y \frac{dy}{dx} = -2x$.
3. Solve: $\frac{dy}{dx} = -\frac{x}{y}$.

At the point $(3,4)$, the slope is $-\frac{3}{4}$, so the tangent line is $y-4 = -\frac{3}{4}(x-3)$.



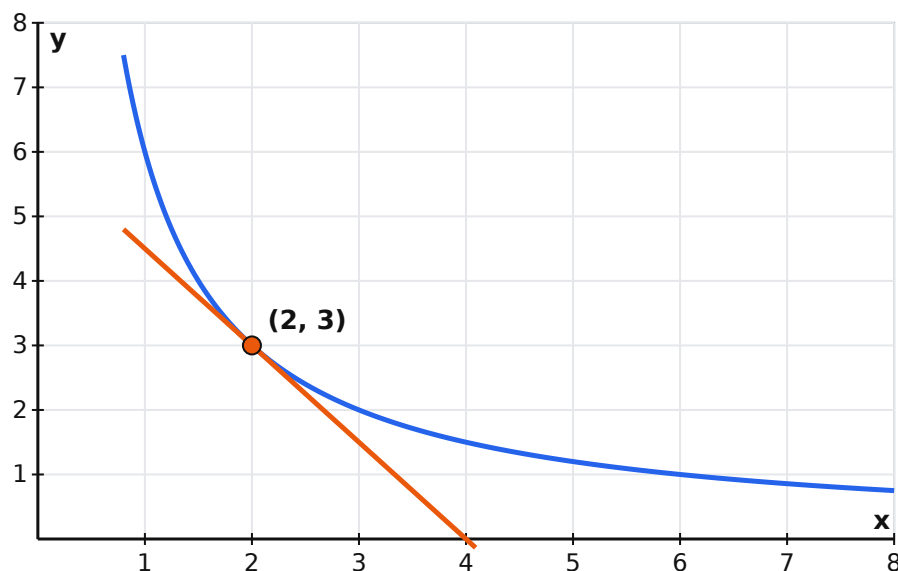
At $(3, 4)$ the slope of the circle is $-x/y = -3/4$.

How do products and other terms work?

For xy , use the product rule: $\frac{d}{dx}[xy] = y + x \frac{dy}{dx}$. For $\sin y$, use the chain rule: $\cos y \cdot \frac{dy}{dx}$. For e^y , the result is $e^y \frac{dy}{dx}$.

When the derivative has y in it, that is normal. Substitute both coordinates of a point to get a numerical slope. The point must satisfy the original equation.

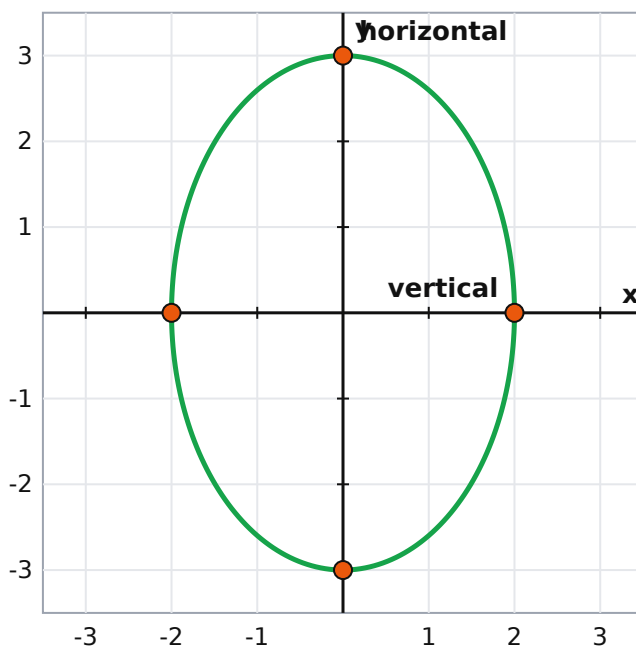
Example with a product. For $xy = 6$, differentiate both sides: $y + x \frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{y}{x}$. At $(2,3)$ the slope is $-\frac{3}{2}$. You can confirm this by solving $y = \frac{6}{x}$ and using the power rule:
 $y' = -\frac{6}{x^2} = -\frac{6}{4} = -\frac{3}{2}$.



The curve $xy = 6$ with its tangent at $(2, 3)$; slope $-3/2$.

When are tangents horizontal or vertical?

A tangent is **horizontal** where the numerator of $\frac{dy}{dx}$ is zero and the denominator is not. It is **vertical** where the denominator is zero and the numerator is not. For $9x^2 + 4y^2 = 36$, $\frac{dy}{dx} = -\frac{9x}{4y}$, so tangents are horizontal at $(0, \pm 3)$ and vertical at $(\pm 2, 0)$.



Horizontal tangents where $dy/dx = 0$; vertical tangents where the denominator is 0.

A common check is to pick a point, compute the slope with implicit differentiation, and compare it with the picture. On the circle, the slope is negative in the first quadrant and

positive in the second quadrant, exactly as the graph shows.

For a second derivative, differentiate $\frac{dy}{dx}$ again, and replace $\frac{dy}{dx}$ with its formula. For the circle, $\frac{d^2y}{dx^2} = -\frac{25}{y^3}$.

Questions about implicit differentiation

What is implicit differentiation used for?

It finds slopes of curves that are not written as $y = f(x)$, such as circles and ellipses, and it handles equations that are hard or impossible to solve for y .

Why do you multiply by dy/dx ?

Because y is a function of x , so differentiating a y expression requires the chain rule. For example, the derivative of y squared with respect to x is $2y$ times dy/dx .

How do you find the slope at a point on an implicit curve?

Find dy/dx in terms of x and y , then substitute both coordinates of the point. Check first that the point satisfies the original equation.

How do you find a vertical tangent?

Write dy/dx as a fraction and find where the denominator is zero while the numerator is nonzero. Then use the original equation to find the matching points.

How do you find a second derivative implicitly?

Differentiate dy/dx with respect to x , apply the chain rule to every y , and then substitute the expression for dy/dx . Simplify using the original equation if possible.