

AP Calculus AB - Implicit Differentiation - Answer Key

Common Core AP CALC AB 3.2 · maths-pdf.fr/en/

This answer key gives a full worked solution for every problem in the implicit differentiation worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 3.2**

Exercise 1 - Circle

Differentiate: $2x + 2y \ y' = 0$, so $y' = -\frac{x}{y}$.

At $(3,4)$: $-\frac{3}{4}$. At $(-3,4)$: $-\frac{-3}{4} = \frac{3}{4}$.

Final answer: $\frac{dy}{dx} = -\frac{x}{y}$; $-\frac{3}{4}$ and $\frac{3}{4}$.

Exercise 2 - Product of x and y

Product rule: $y + x \ y' = 0$, so $y' = -\frac{y}{x}$.

Final answer: $-\frac{y}{x}$.

Exercise 3 - Sum of cubes

$3x^2 + 3y^2 y' = 0$, so $y' = -\frac{x^2}{y^2}$.

Final answer: $-\frac{x^2}{y^2}$.

Exercise 4 - Mixed terms

Differentiate: $2x + (y + xy') + 2yy' = 0$.

Collect: $y'(x + 2y) = -(2x + y)$.

Final answer: $-\frac{2x+y}{x+2y}$.

Exercise 5 - Sine of y

$$\cos y \cdot y' = 1, \text{ so } y' = \frac{1}{\cos y} = \sec y.$$

Final answer: $\sec y$.

Exercise 6 - Exponential in y

$$e^y y' = 1 + y'. \text{ Collect: } y'(e^y - 1) = 1.$$

Final answer: $\frac{1}{e^y - 1}$.

Exercise 7 - Semicubical parabola

$$2yy' = 3x^2.$$

Final answer: $\frac{3x^2}{2y}$.

Exercise 8 - Product and a cube

$$\text{Differentiate: } 2xy + x^2y' + 3y^2y' = 0.$$

$$\text{Collect: } y'(x^2 + 3y^2) = -2xy.$$

Final answer: $-\frac{2xy}{x^2 + 3y^2}$.

Exercise 9 - Slope at a point

Check: $4+2+1=7$. From the earlier formula, $y' = -\frac{2x+y}{x+2y}$.

At (2,1): $-\frac{4+1}{2+2} = -\frac{5}{4}$.

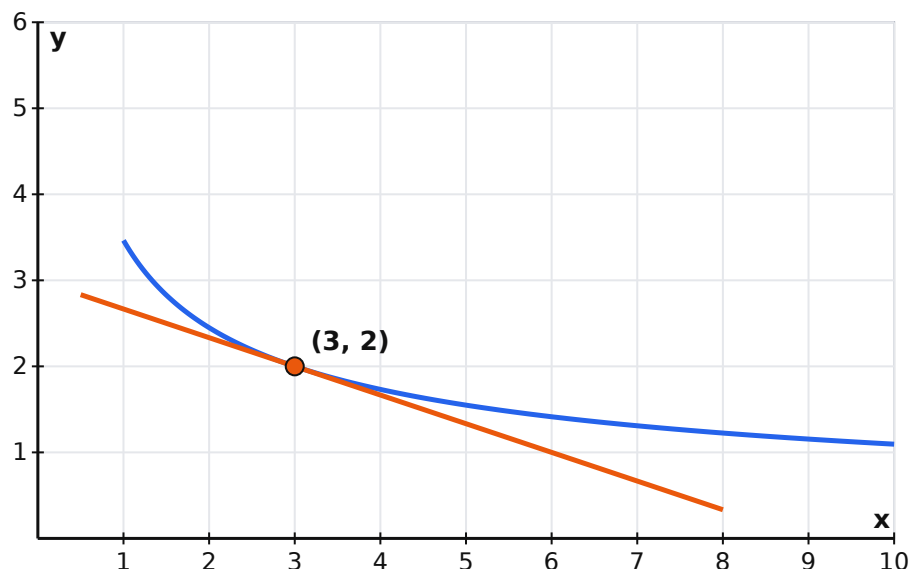
Final answer: $-\frac{5}{4}$.

Exercise 10 - Tangent line to an implicit curve

Check: $3 \cdot 4 = 12$. Differentiate: $y^2 + 2xyy' = 0$, so $y' = -\frac{y}{2x}$.

At $(3, 2)$: $-\frac{2}{3} = -\frac{1}{3}$. Tangent: $y - 2 = -\frac{1}{3}(x - 3)$.

Final answer: $y = -\frac{1}{3}x + 3$.



The tangent at $(3, 2)$ has slope $-1/3$.

Exercise 11 - Horizontal and vertical tangents

$18x + 8yy' = 0$, so $y' = -\frac{9x}{4y}$.

Horizontal: numerator zero, $x = 0$, so $4y^2 = 36$ and $y = \pm 3$.

Vertical: denominator zero, $y = 0$, so $9x^2 = 36$ and $x = \pm 2$.

Final answer: horizontal at $(0, 3)$, $(0, -3)$; vertical at $(2, 0)$, $(-2, 0)$.

Exercise 12 - A shifted circle

Differentiate: $2x + 2yy' - 4 + 6y' = 0$, so $y' = \frac{4 - 2x}{2y + 6} = \frac{2 - x}{y + 3}$.

Horizontal: $2 - x = 0$, so $x = 2$. Substitute: $4 + y^2 - 8 + 6y = 12$, so $y^2 + 6y - 16 = 0$ and $(y + 8)(y - 2) = 0$.

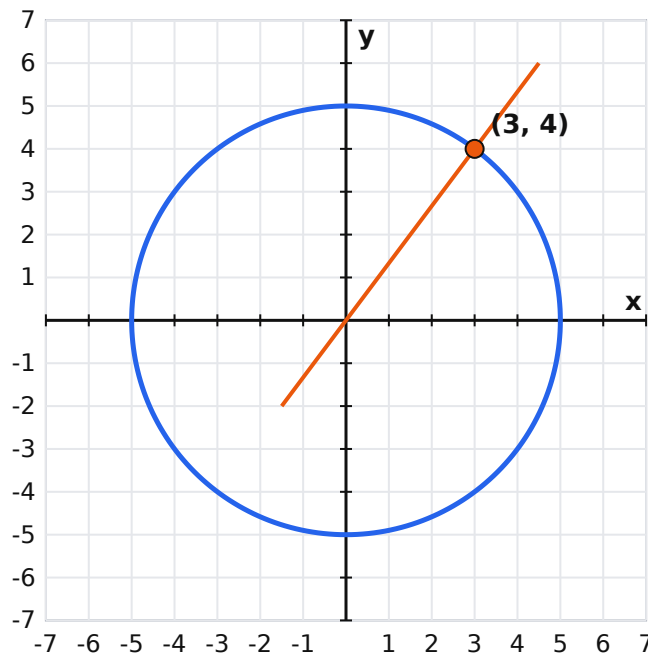
Final answer: $(2, 2)$ and $(2, -8)$.

Exercise 13 - Normal line

The tangent slope is $-\frac{3}{4}$, so the normal slope is the negative reciprocal $\frac{4}{3}$.

$y-4=\frac{4}{3}(x-3)$ simplifies to $y=\frac{4}{3}x$, a line through the center.

Final answer: $y=\frac{4}{3}x$.



The normal line passes through the center.

Exercise 14 - Logarithm in the equation

Rewrite: $\ln x + \ln y = x$. Differentiate: $\frac{1}{x} + \frac{y'}{y} = 1$.

Solve: $\frac{y'}{y} = 1 - \frac{1}{x}$, so $y' = y\left(1 - \frac{1}{x}\right)$.

Final answer: $y\left(1 - \frac{1}{x}\right)$.

Exercise 15 - Sine of a product

Chain and product rules: $\cos(xy)(y + xy') = 1$.

So $y + xy' = \sec(xy)$ and $y' = \frac{\sec(xy) - y}{x}$.

Final answer: $\frac{\sec(xy) - y}{x}$.

Exercise 16 - Second derivative

From $y' = -\frac{x}{y}$, differentiate with the quotient rule: $y'' = -\frac{y-x y'}{y^2}$.

Substitute $y' = -\frac{x}{y}$: $y'' = -\frac{y+\frac{x^2}{y}}{y^2} = -\frac{y^2+x^2}{y^3} = -\frac{25}{y^3}$.

Final answer: $-\frac{25}{y^3}$.

Exercise 17 - Vertical tangents

$2x - y - xy' + 2yy' = 0$, so $y' = \frac{y-2x}{2y-x}$.

Vertical when $2y-x=0$, that is $x=2y$. Substitute: $4y^2-2y^2+y^2=3$, so $y^2=1$.

Final answer: $(2,1)$ and $(-2,-1)$.

Exercise 18 - Drone flight path

$y' = -\frac{x}{y} = -\frac{6}{8} = -\frac{3}{4}$.

$y-8 = -\frac{3}{4}(x-6)$, which is $y = -\frac{3}{4}x + \frac{25}{2}$.

Final answer: slope $-\frac{3}{4}$; $y = -\frac{3}{4}x + 12.5$.

Exercise 19 - Running track

Check: $4 \cdot \frac{81}{25} + 9 \cdot \frac{64}{25} = \frac{324+576}{25} = 36$.

$8x + 18yy' = 0$, so $y' = -\frac{4x}{9y} = -\frac{4 \cdot 9/5}{9 \cdot 8/5} = -\frac{1}{2}$.

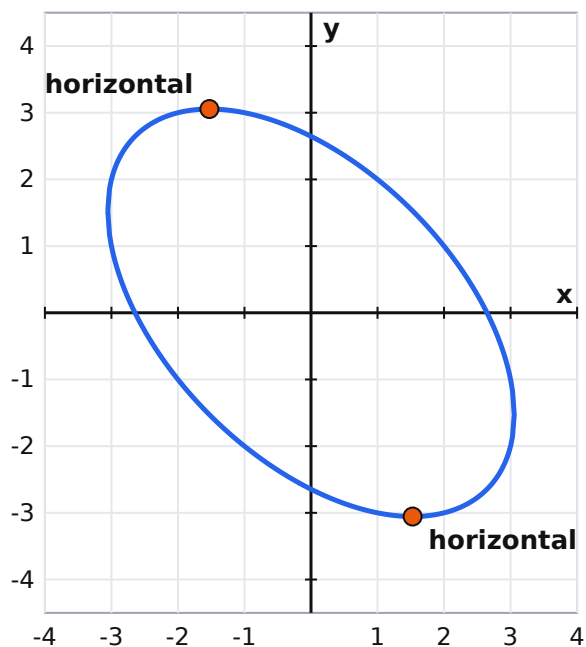
Final answer: $-\frac{1}{2}$.

Exercise 20 - Horizontal tangent of a tilted curve

From $y' = -\frac{2x+y}{x+2y}$, the tangent is horizontal when $2x+y=0$, so $y=-2x$.

Substitute: $x^2-2x^2+4x^2=7$, so $3x^2=7$ and $x=\pm\sqrt{\frac{7}{3}}$.

Final answer: $\left(\sqrt{\frac{7}{3}}, -2\sqrt{\frac{7}{3}}\right)$ and $\left(-\sqrt{\frac{7}{3}}, 2\sqrt{\frac{7}{3}}\right)$.



Horizontal tangents at $x = \pm\sqrt{7/3}$.

Exercise 21 - Derivative of a fractional power

$$3y^2y' = 2x, \text{ so } y' = \frac{2x}{3y^2}.$$

$$\text{Substitute } y = x^{2/3}: y' = \frac{2x}{3x^{4/3}} = \frac{2}{3}x^{-1/3}.$$

The power rule gives $\frac{2}{3}x^{2/3-1} = \frac{2}{3}x^{-1/3}$. They agree.

$$\text{Final answer: } \frac{2x}{3y^2} = \frac{2}{3}x^{-1/3}.$$