

# AP Calculus AB - Implicit Differentiation - Worksheet

Common Core AP CALC AB 3.2 · [maths-pdf.fr/en/](https://maths-pdf.fr/en/)

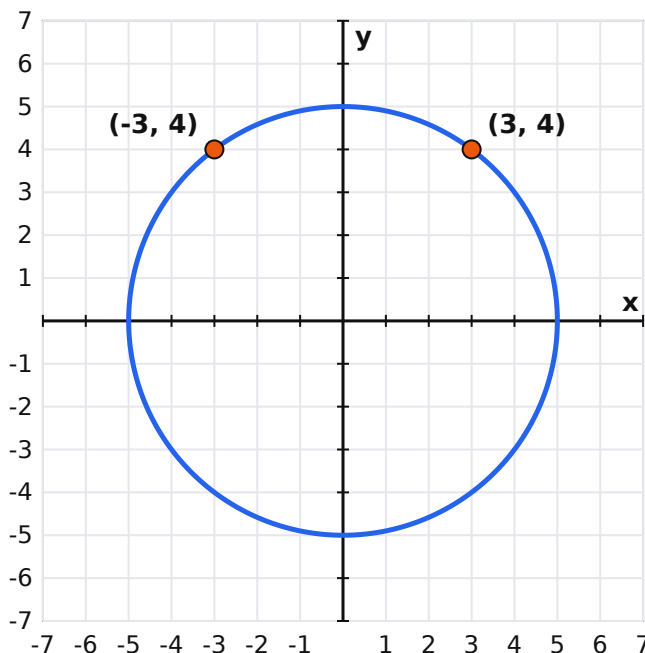
Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_/\_\_\_\_/\_\_\_\_

These 21 problems practice implicit differentiation, slopes of curves, tangent lines, horizontal and vertical tangents and second derivatives (AP Calculus AB, topics 3.2). The early problems build the basic skill, and each one uses a different function or situation so that no two feel alike.

Later problems use circles, ellipses and tilted curves, vertical tangents, second derivatives and situations such as a drone path and a running track. Work in pencil, show every step, and check the answer key only after a full attempt.

**AP Calculus AB** · Common Core State Standards: **AP CALC AB 3.2**

## Exercise 1 - Circle



The circle of radius 5.

Find  $\frac{dy}{dx}$  for  $x^2 + y^2 = 25$  and then the slope at  $(3,4)$  and at  $(-3,4)$ .

### Exercise 2 - Product of x and y

Find  $\frac{dy}{dx}$  for  $xy = 6$ .

### Exercise 3 - Sum of cubes

Find  $\frac{dy}{dx}$  for  $x^3 + y^3 = 9$ .

### Exercise 4 - Mixed terms

Find  $\frac{dy}{dx}$  for  $x^2 + xy + y^2 = 7$ .

### Exercise 5 - Sine of y

Find  $\frac{dy}{dx}$  for  $\sin y = x$ .

### Exercise 6 - Exponential in y

Find  $\frac{dy}{dx}$  for  $e^y = x + y$ .

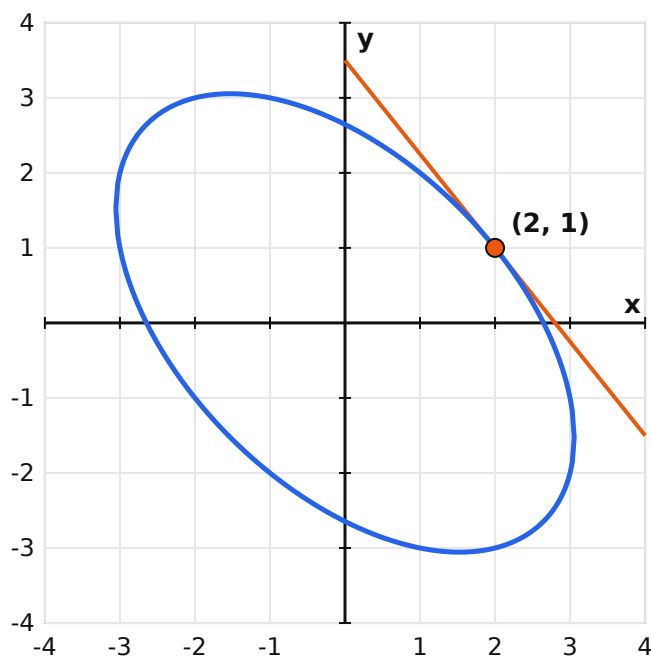
### Exercise 7 - Semicubical parabola

Find  $\frac{dy}{dx}$  for  $y^2 = x^3$ .

### Exercise 8 - Product and a cube

Find  $\frac{dy}{dx}$  for  $x^2y + y^3 = 10$ .

### Exercise 9 - Slope at a point



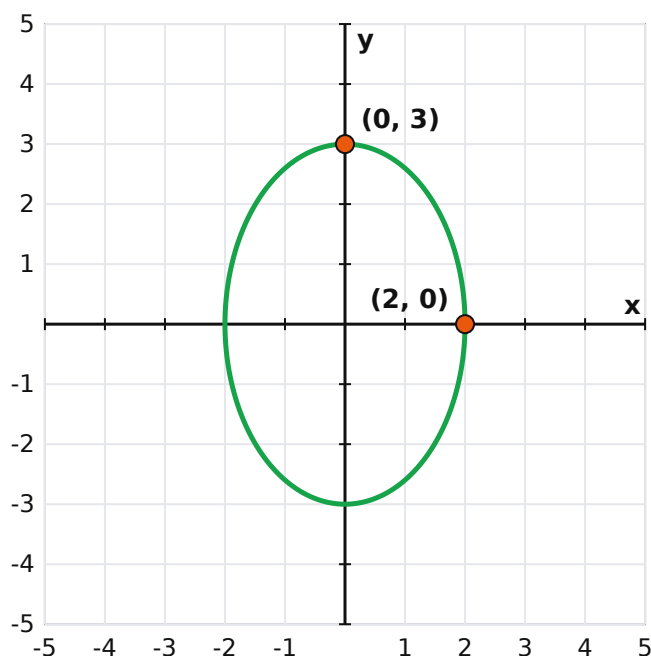
The tangent at (2, 1) has slope  $-5/4$ .

The point (2,1) lies on  $x^2+xy+y^2=7$ . Find the slope of the curve there.

### Exercise 10 - Tangent line to an implicit curve

Find the tangent line to  $xy^2=12$  at (3,2).

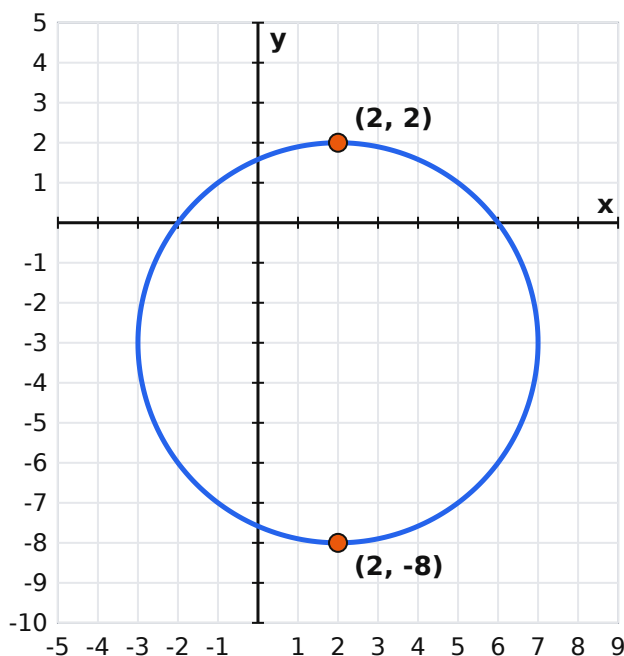
### Exercise 11 - Horizontal and vertical tangents



The ellipse  $9x^2 + 4y^2 = 36$ .

For the ellipse  $9x^2 + 4y^2 = 36$ , find  $\frac{dy}{dx}$  and the points with horizontal and vertical tangents.

### Exercise 12 - A shifted circle



The circle  $x^2 + y^2 - 4x + 6y = 12$ .

Find the points where  $x^2 + y^2 - 4x + 6y = 12$  has a horizontal tangent.

### Exercise 13 - Normal line

Find the normal line to  $x^2 + y^2 = 25$  at  $(3, 4)$ .

### Exercise 14 - Logarithm in the equation

Find  $\frac{dy}{dx}$  for  $\ln(xy) = x$ .

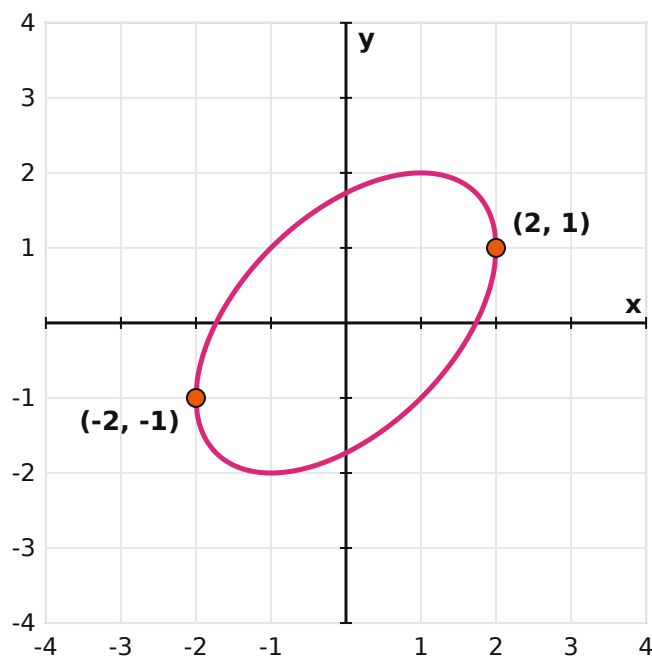
### Exercise 15 - Sine of a product

Find  $\frac{dy}{dx}$  for  $\sin(xy) = x$ .

### Exercise 16 - Second derivative

Find  $\frac{d^2y}{dx^2}$  for  $x^2 + y^2 = 25$ .

### Exercise 17 - Vertical tangents



The curve  $x^2 - xy + y^2 = 3$ .

Find the points on  $x^2 - xy + y^2 = 3$  where the tangent is vertical.

### Exercise 18 - Drone flight path

A drone flies along the circle  $x^2 + y^2 = 100$  (distances in miles from the base). At the point  $(6, 8)$ , find the slope of its path and the equation of the line it would follow if it flew straight.

### Exercise 19 - Running track

A model of an elliptical running track is  $4x^2 + 9y^2 = 36$  (in hundreds of feet). Verify that  $(\frac{9}{5}, \frac{8}{5})$  is on the track and find the slope there.

### Exercise 20 - Horizontal tangent of a tilted curve

Find the points where  $x^2 + xy + y^2 = 7$  has a horizontal tangent.

### Exercise 21 - Derivative of a fractional power

Differentiate  $y^3 = x^2$  implicitly and show that the result agrees with the power rule for  $y = x^{2/3}$ .