

AP Calculus AB - Implicit Differentiation - Test Answer Key

Common Core AP CALC AB 3.2 · maths-pdf.fr/en/

Use this answer key to score the implicit differentiation test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

AP Calculus AB · Common Core State Standards: **AP CALC AB 3.2**

Question 1 - Implicit derivative [3 pts]

Differentiate: $3x^2 + 3y + 3xy' + 2yy' = 0$.

Collect: $y'(3x + 2y) = -(3x^2 + 3y)$.

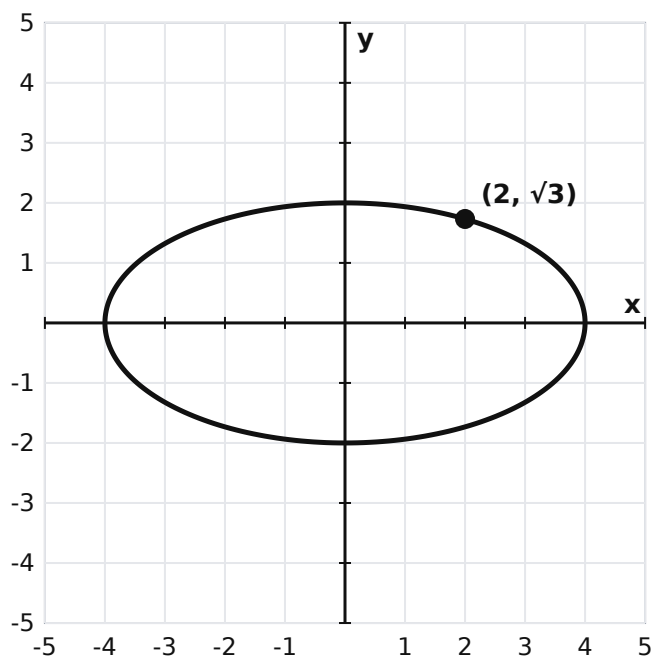
Final answer: $\frac{dy}{dx} = -\frac{3x^2 + 3y}{3x + 2y}$.

Scoring: 1 point for each term differentiated correctly (product and chain); 1 point for solving.

Question 2 - Slope on an ellipse [2 pts]

$2x + 8yy' = 0$, so $y' = -\frac{x}{4y}$. At the point: $-\frac{2}{4\sqrt{3}} = -\frac{1}{2\sqrt{3}}$.

Tangent: $y - \sqrt{3} = -\frac{1}{2\sqrt{3}}(x - 2)$.



The ellipse $x^2 + 4y^2 = 16$.

Scoring: 1 point for the derivative and the slope; 1 point for the equation.

Question 3 - Chain rule in y [2 pts]

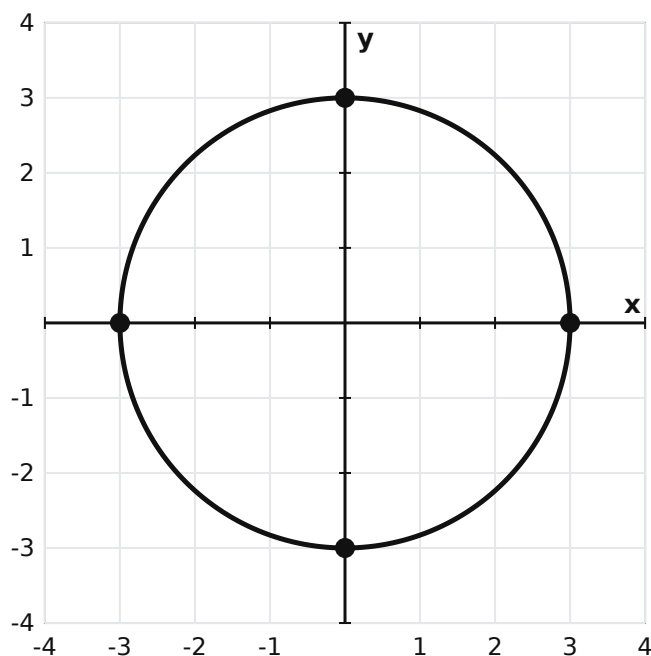
$-\sin y \cdot y' = 2xy + x^2 y'$. Collect: $y'(-\sin y - x^2) = 2xy$.

Final answer: $\frac{dy}{dx} = -\frac{2xy}{x^2 + \sin y}$.

Scoring: 1 point for the differentiation; 1 point for solving.

Question 4 - Horizontal and vertical tangents [3 pts]

$y' = -\frac{x}{y}$. Horizontal: $x=0$, so $(0,3)$ and $(0,-3)$. Vertical: $y=0$, so $(3,0)$ and $(-3,0)$.



The circle $x^2 + y^2 = 9$.

Scoring: 1 point for the derivative; 1 point for the horizontal points; 1 point for the vertical points.

Question 5 - Tangent line [3 pts]

Differentiate: $2x + y + xy' = 0$, so $y' = -\frac{2x+y}{x}$. At $(1,1)$: -3 .

Tangent: $y - 1 = -3(x - 1)$. Final answer: $y = -3x + 4$.

Scoring: 1 point for the derivative; 1 point for the slope; 1 point for the equation.

Question 6 - Second derivative [2 pts]

$y' = -\frac{x}{y}$. Then $y'' = -\frac{y - xy'}{y^2} = -\frac{y + \frac{x^2}{y}}{y^2} = -\frac{y^2 + x^2}{y^3} = -\frac{25}{y^3}$.

Scoring: 1 point for the quotient rule and substituting y' ; 1 point for using the original equation.

Question 7 - Implicit slope with a product [2 pts]

$2x + y + xy' = 0$ gives $y' = -\frac{2x+y}{x}$. At $(1,1)$: -3 .



Part of the curve $x^2 + xy = 2$.

Scoring: 1 point for the product rule term; 1 point for the slope.

Question 8 - Ferris wheel [3 pts]

- $\frac{dy}{dx} = -\frac{x}{y}$.
- $-\frac{4\sqrt{3}}{4} = -\sqrt{3}$.
- $-\frac{0}{8} = 0$: the path is horizontal at the top.

Scoring: 1 point per part.

Total

20 points