

AP Calculus AB - Derivatives of Inverse Functions - Answer Key

Common Core AP CALC AB 3.3, AP CALC AB 3.4, AP CALC AB 3.5, AP CALC AB 3.6 · maths-pdf.fr/en/

This answer key gives a full worked solution for every problem in the inverse derivatives worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 3.3, AP CALC AB 3.4, AP CALC AB 3.5, AP CALC AB 3.6**

Exercise 1 - Inverse derivative from values

Since $f(2)=5$, $f^{-1}(5)=2$. Then $(f^{-1})'(5)=\frac{1}{f'(2)}=\frac{1}{4}$.

Final answer: $\frac{1}{4}$.

Exercise 2 - Inverse of a cubic

Find α with $f(\alpha)=3$: $\alpha=1$ since $1+1+1=3$.

$f'(x)=3x^2+1$, so $f'(1)=4$.

$(f^{-1})'(3)=\frac{1}{4}$.

Final answer: $\frac{1}{4}$.

Exercise 3 - Inverse with sine

$f(0)=0$, so $\alpha=0$. $f'(x)=2+\cos x$, so $f'(0)=3$.

$(f^{-1})'(0)=\frac{1}{3}$.

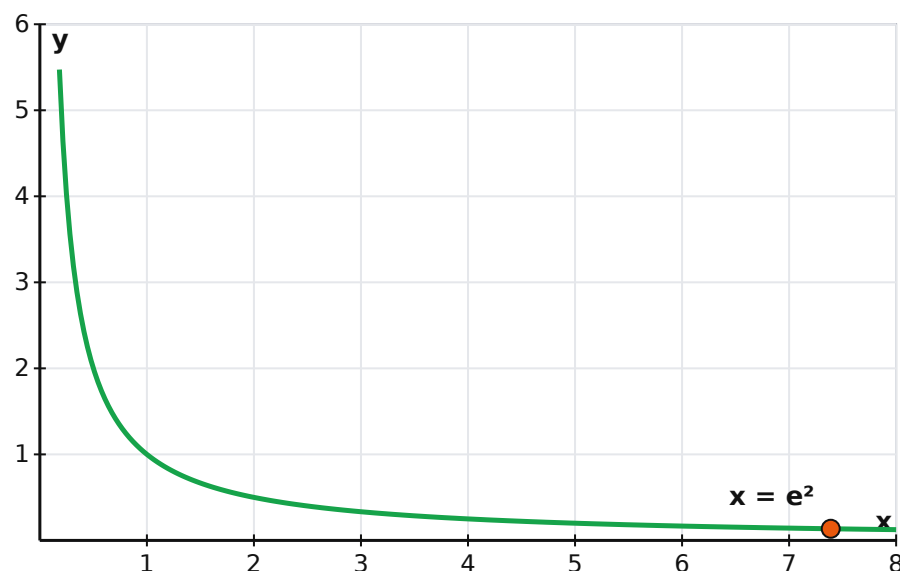
Final answer: $\frac{1}{3}$.

Exercise 4 - Inverse of the exponential

Here $f(2) = e^2$ and $f'(2) = e^2$. So $(f^{-1})'(e^2) = \frac{1}{e^2}$.

This agrees with $\frac{d}{dx}[\ln x] = \frac{1}{x}$ at $x = e^2$.

Final answer: $\frac{1}{e^2}$.

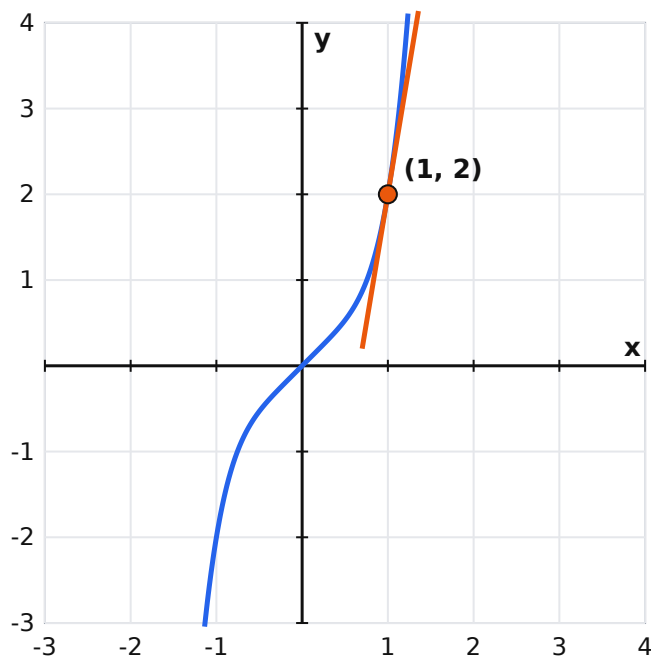


The derivative $1/x$ of $\ln x$ at $x = e^2$ is $1/e^2$.

Exercise 5 - Inverse of a fifth-degree polynomial

$f(1) = 2$, so $a = 1$. $f'(x) = 5x^4 + 1$ gives $f'(1) = 6$.

Final answer: $(f^{-1})'(2) = \frac{1}{6}$.



The tangent at (1, 2) has slope 6, so the inverse has slope $\frac{1}{6}$ at (2, 1).

Exercise 6 - Inverse from a table

$$f(1) = 7, \text{ so } (f^{-1})'(7) = \frac{1}{f'(1)} = \frac{1}{3}.$$

$$f(3) = 12, \text{ so } (f^{-1})'(12) = \frac{1}{f'(3)} = \frac{1}{6}.$$

Final answer: $\frac{1}{3}$ and $\frac{1}{6}$.

Exercise 7 - Arcsine at a point

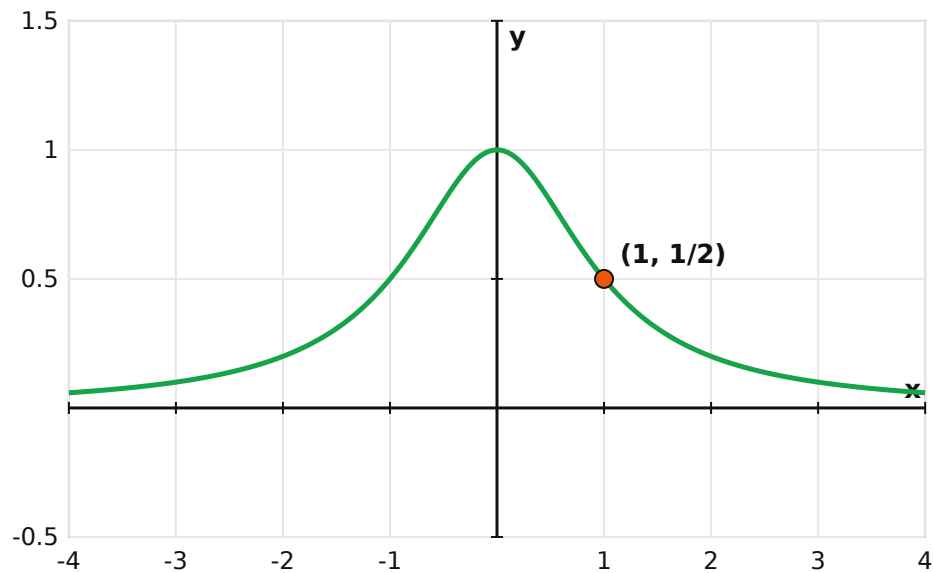
$$\frac{1}{\sqrt{1-x^2}}. \text{ At } x = \frac{1}{2}: \frac{1}{\sqrt{1-\frac{1}{4}}} = \frac{1}{\sqrt{3/2}} = \frac{2}{\sqrt{3}}.$$

Final answer: $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$.

Exercise 8 - Arctangent at a point

$$\frac{dy}{dx} = \frac{1}{1+x^2}, \text{ so at } x=1 \text{ the slope is } \frac{1}{1+1} = \frac{1}{2}.$$

Final answer: $\frac{1}{2}$.



The derivative of $\arctan x$ is $1/(1 + x^2)$.

Exercise 9 - Arcsine with a chain

Outer $\sin^{-1}u$, inner $3x$: $\frac{1}{\sqrt{1-(3x)^2}} \cdot 3$.

Final answer: $f'(x) = \frac{3}{\sqrt{1-9x^2}}$.

Exercise 10 - Arctangent of a square

$\frac{1}{1+(x^2)^2} \cdot 2x$.

Final answer: $f'(x) = \frac{2x}{1+x^4}$.

Exercise 11 - Arccosine of x over 2

$-\frac{1}{\sqrt{1-\frac{x^2}{4}}} \cdot \frac{1}{2} = -\frac{1}{2\sqrt{4-x^2}} = -\frac{1}{\sqrt{4-x^2}}$.

Final answer: $f'(x) = -\frac{1}{\sqrt{4-x^2}}$.

Exercise 12 - Product with arctangent

Product rule: $1 \cdot \tan^{-1}x + x \cdot \frac{1}{1+x^2}$.

Final answer: $f'(x) = \tan^{-1}x + \frac{x}{1+x^2}$.

Exercise 13 - Exponential times arcsine

Product rule: $e^x \sin^{-1}x + e^x \cdot \frac{1}{\sqrt{1-x^2}}$.

Final answer: $f'(x) = e^x \left(\sin^{-1}x + \frac{1}{\sqrt{1-x^2}} \right)$.

Exercise 14 - Arctangent of an exponential

$\frac{1}{1+(e^x)^2} \cdot e^x$.

Final answer: $f'(x) = \frac{e^x}{1+e^{2x}}$.

Exercise 15 - Three derivatives

$f'(x) = 4x^3 - 9x^2 + 2$, $f''(x) = 12x^2 - 18x$, $f'''(x) = 24x - 18$.

$f''(1) = 12 - 18 = -6$.

Final answer: $f''(1) = -6$.

Exercise 16 - Second derivative of x e to the x

Product rule: $f' = e^x + x e^x = (x+1)e^x$.

Again: $f'' = e^x + (x+1)e^x = (x+2)e^x$.

Final answer: $f''(x) = (x+2)e^x$.

Exercise 17 - A pattern of derivatives

$f' = 3e^{3x}$, $f'' = 9e^{3x}$, $f''' = 27e^{3x}$. Each derivative multiplies by 3.

Final answer: $f^{(4)}(x) = 81e^{3x}$.

Exercise 18 - Second derivative of a log

$$f' = x^{-1}, \text{ so } f'' = -x^{-2}.$$

$$\text{Final answer: } -\frac{1}{x^2}.$$

Exercise 19 - Velocity and acceleration

$$v(t) = 3t^2 - 18t + 15 = 3(t-1)(t-5) \text{ and } a(t) = 6t - 18.$$

$$\text{At rest: } t=1 \text{ and } t=5. \text{ Accelerations: } a(1) = -12 \text{ and } a(5) = 12 \text{ ft/s}^2.$$

$$\text{Final answer: } t=1, 5; a(1) = -12, a(5) = 12 \text{ ft/s}^2.$$

Exercise 20 - Choosing the rule

a. Product rule: $2x \sin x + x^2 \cos x$.

b. Quotient rule: $\frac{2x(x-1) - (x^2+1)}{(x-1)^2} = \frac{x^2-2x-1}{(x-1)^2}$.

c. Chain rule: $\frac{6x}{2\sqrt{3x^2+1}} = \frac{3x}{\sqrt{3x^2+1}}$.

d. Term by term: $3x^2 - 5$.

Exercise 21 - Implicit second derivative

$$y + xy' = 0, \text{ so } y' = -\frac{y}{x}.$$

$$\text{Quotient rule: } y'' = -\frac{y'x - y}{x^2} = -\frac{-y - y}{x^2} = \frac{2y}{x^2}.$$

$$\text{With } y = \frac{1}{x}: \frac{2}{x^3}. \text{ Check: } \frac{d^2}{dx^2}[x^{-1}] = 2x^{-3}. \text{ They match.}$$

$$\text{Final answer: } y'' = \frac{2y}{x^2} = \frac{2}{x^3}.$$

Exercise 22 - Volume and radius

$$V(3) = \frac{4}{3}\pi \cdot 27 = 36\pi, \text{ so the radius is 3 inches. } V'(r) = 4\pi r^2, \text{ so } V'(3) = 36\pi.$$

$$r'(36\pi) = \frac{1}{V'(3)} = \frac{1}{36\pi}.$$

Final answer: $\frac{1}{36\pi} \approx 0.0088 \text{ in/in}^3$: when the balloon holds 36π cubic inches, each extra cubic inch of air adds about 0.0088 inch of radius.

Exercise 23 - Temperature scales

$F'(C) = 1.8$ for every C . The inverse derivative is $\frac{1}{1.8} = \frac{5}{9}$.

Final answer: $\frac{5}{9}$.

A change of 1 °F corresponds to a change of $\frac{5}{9}$ °C.