

AP Calculus AB - Derivatives of Inverse Functions - Worksheet

Common Core AP CALC AB 3.3, AP CALC AB 3.4, AP CALC AB 3.5, AP CALC AB 3.6 · maths-pdf.fr/en/

Name: _____ Class: _____ Date: ____/____/____

These 23 problems practice derivatives of inverse functions, derivatives of inverse trigonometric functions, choosing differentiation methods and higher-order derivatives (AP Calculus AB, topics 3.3, 3.4, 3.5, 3.6). The early problems build the basic skill, and each one uses a different function or situation so that no two feel alike.

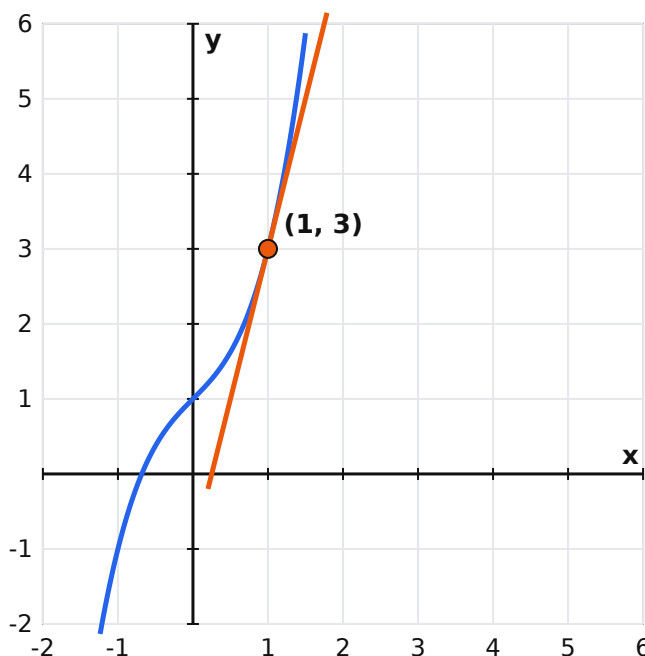
Later problems cover tables of values, products with inverse trig functions, second derivatives and situations such as a balloon volume and a cart on a track. Work in pencil, show every step, and check the answer key only after a full attempt.

AP Calculus AB · Common Core State Standards: **AP CALC AB 3.3, AP CALC AB 3.4, AP CALC AB 3.5, AP CALC AB 3.6**

Exercise 1 - Inverse derivative from values

A function has $f(2)=5$ and $f'(2)=4$. Find $(f^{-1})'(5)$.

Exercise 2 - Inverse of a cubic



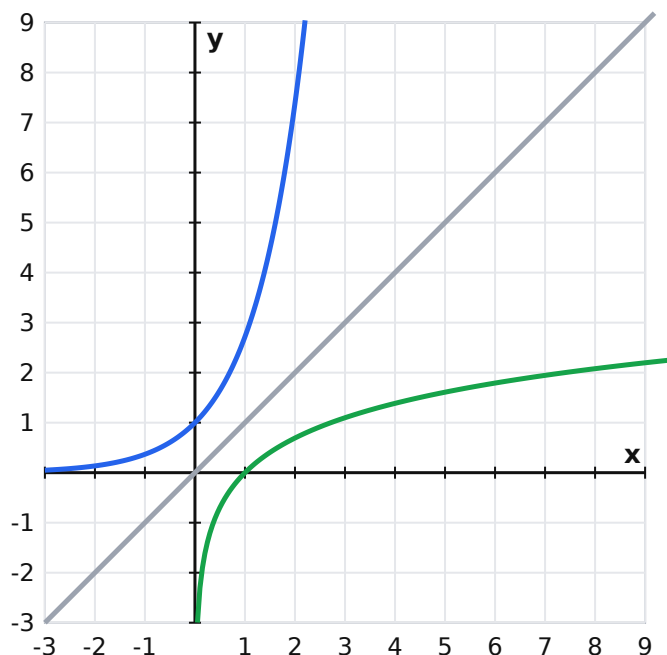
The graph of $f(x) = x^3 + x + 1$.

Let $f(x) = x^3 + x + 1$. Find $(f^{-1})'(3)$.

Exercise 3 - Inverse with sine

Let $f(x) = 2x + \sin x$. Find $(f^{-1})'(0)$.

Exercise 4 - Inverse of the exponential



e^x and $\ln x$ are inverses.

Let $f(x) = e^x$. Find $(f^{-1})'(e^2)$ and compare with $\frac{d}{dx}[\ln x]$.

Exercise 5 - Inverse of a fifth-degree polynomial

Let $f(x) = x^5 + x$. Find $(f^{-1})'(2)$.

Exercise 6 - Inverse from a table

x	0	1	2	3
f(x)	4	7	9	12
f'(x)	2	3	5	6

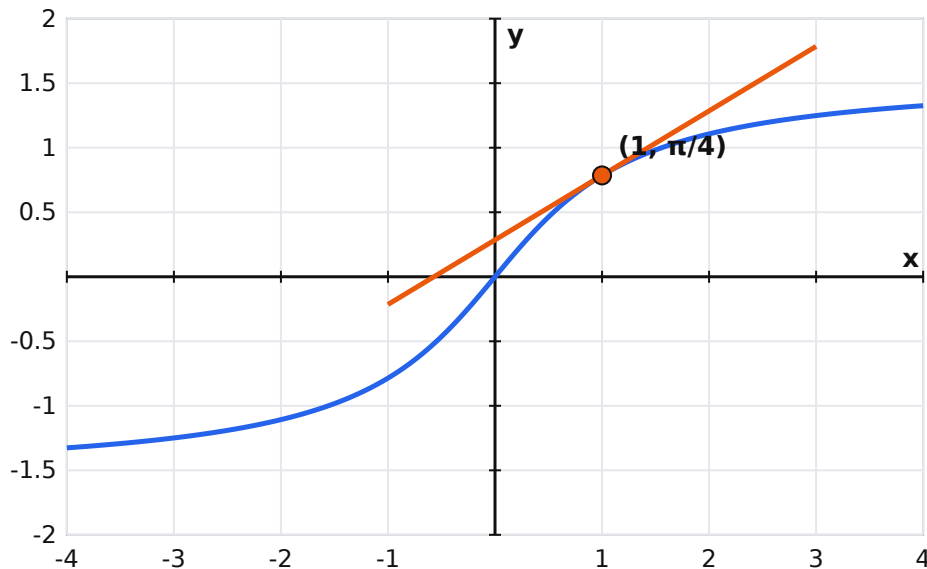
Values of f and f'.

Use the table to find $(f^{-1})'(7)$ and $(f^{-1})'(12)$.

Exercise 7 - Arcsine at a point

Find $\frac{d}{dx}[\sin^{-1}x]$ and evaluate it at $x = \frac{1}{2}$.

Exercise 8 - Arctangent at a point



The slope of $\arctan x$ at $x = 1$ is $1/2$.

Find the slope of $y = \tan^{-1}x$ at $x = 1$.

Exercise 9 - Arcsine with a chain

Differentiate $f(x) = \sin^{-1}(3x)$.

Exercise 10 - Arctangent of a square

Differentiate $f(x) = \tan^{-1}(x^2)$.

Exercise 11 - Arccosine of x over 2

Differentiate $f(x) = \cos^{-1}\left(\frac{x}{2}\right)$.

Exercise 12 - Product with arctangent

Differentiate $f(x) = x \tan^{-1}x$.

Exercise 13 - Exponential times arcsine

Differentiate $f(x) = e^x \sin^{-1} x$.

Exercise 14 - Arctangent of an exponential

Differentiate $f(x) = \tan^{-1}(e^x)$.

Exercise 15 - Three derivatives

For $f(x) = x^4 - 3x^3 + 2x$, find f' , f'' , f''' and $f''(1)$.

Exercise 16 - Second derivative of $x e^x$

Find $f''(x)$ for $f(x) = x e^x$.

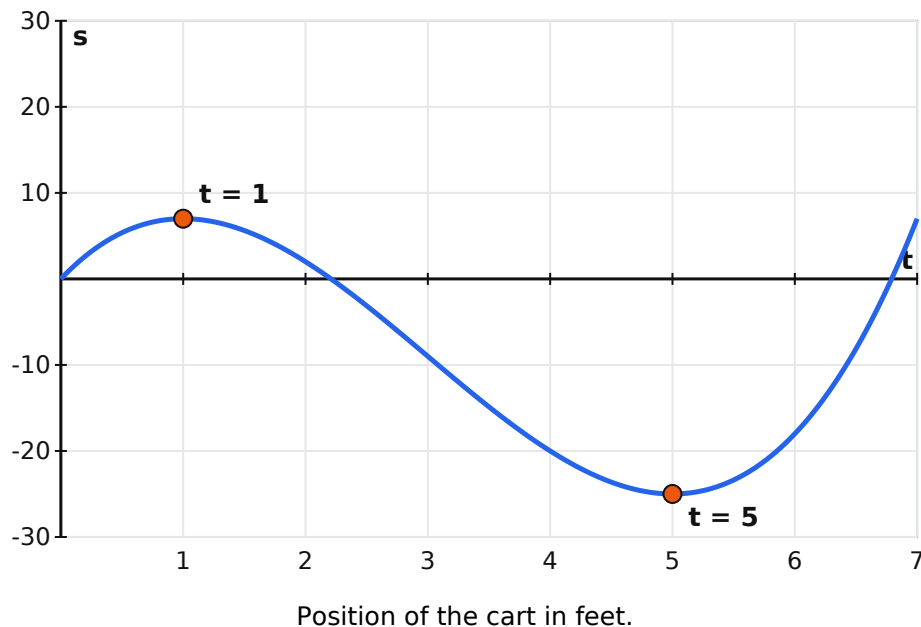
Exercise 17 - A pattern of derivatives

Find the fourth derivative of $f(x) = e^{3x}$ by finding the first few derivatives and spotting the pattern.

Exercise 18 - Second derivative of a log

Find $f''(x)$ for $f(x) = \ln x$.

Exercise 19 - Velocity and acceleration



A cart on a track has position $s(t) = t^3 - 9t^2 + 15t$ feet after t seconds. Find $v(t)$, $a(t)$, the times when the cart is at rest, and the acceleration at those times.

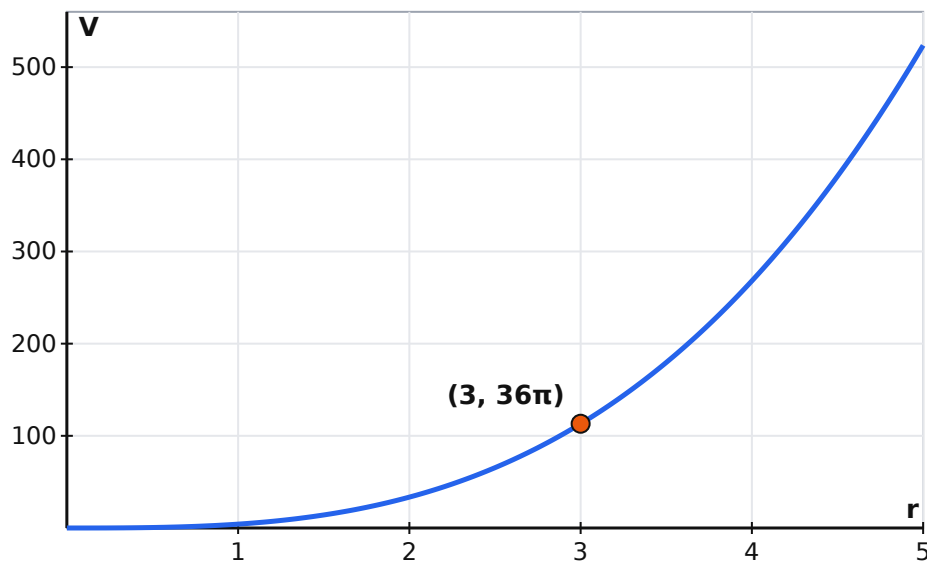
Exercise 20 - Choosing the rule

Name the best rule and differentiate: (a) $x^2 \sin x$ (b) $\frac{x^2+1}{x-1}$ (c) $\sqrt{3x^2+1}$ (d) x^3-5x .

Exercise 21 - Implicit second derivative

For $xy=1$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ implicitly, and check with $y=\frac{1}{x}$.

Exercise 22 - Volume and radius



Volume of a sphere (cubic inches) as a function of radius (inches).

The volume of a balloon in cubic inches is $V(r) = \frac{4}{3}\pi r^3$. Let $r(V)$ be the inverse function. Find $r'(36\pi)$ and interpret it.

Exercise 23 - Temperature scales

A converter uses $F(C) = 1.8C + 32$. Find $(F^{-1})'(50)$ and explain what it means.