

# AP Calculus AB - Derivatives of Inverse Functions - Test Answer Key

Common Core AP CALC AB 3.3, AP CALC AB 3.4, AP CALC AB 3.5, AP CALC AB 3.6 · [maths-pdf.fr/en/](https://maths-pdf.fr/en/)

Use this answer key to score the inverse derivatives test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

**AP Calculus AB** · Common Core State Standards: **AP CALC AB 3.3, AP CALC AB 3.4, AP CALC AB 3.5, AP CALC AB 3.6**

## Question 1 - Inverse from a table [3 pts]

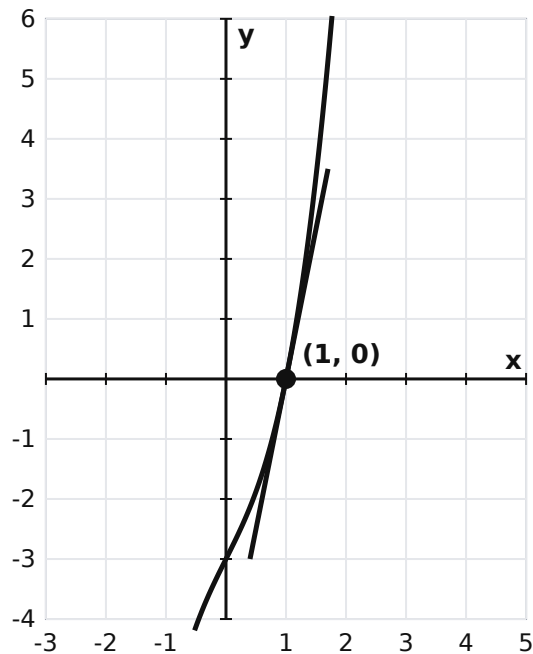
- a.  $f(2)=9$ , so  $\frac{1}{f'(2)}=\frac{1}{5}$ .
- b.  $f(3)=15$ , so  $\frac{1}{f'(3)}=\frac{1}{8}$ .
- c.  $f(1)=4$ , so  $f^{-1}(4)=1$ .

*Scoring: 1 point per part.*

## Question 2 - Inverse of a cubic [2 pts]

$$f(1)=0, \text{ so } a=1. \quad f'(x)=3x^2+2, \text{ so } f'(1)=5.$$

Final answer:  $\frac{1}{5}$ .



Graph of  $f(x) = x^3 + 2x - 3$ .

Scoring: 1 point for finding  $a$  and  $f'(a)$ ; 1 point for the reciprocal.

### Question 3 - Inverse trig derivatives [2 pts]

$$g'(x) = \frac{1}{\sqrt{1-x^2}} + \frac{2}{1+4x^2}.$$

Scoring: 1 point per term.

### Question 4 - Inverse trig and a root [2 pts]

$$h'(x) = \frac{1}{1+(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}(1+x)}.$$

Scoring: 1 point for the arctangent derivative; 1 point for the chain factor.

### Question 5 - Second derivative [3 pts]

$$f'(x) = 5x^4 - 6x^2 + \cos x \text{ and } f''(x) = 20x^3 - 12x - \sin x.$$

$$f''(0) = 0 - 0 - 0 = 0.$$

Scoring: 1 point for  $f'$ ; 1 point for  $f''$ ; 1 point for the value.

### Question 6 - Acceleration [2 pts]

$$v = 6t^2 - 30t + 36 = 6(t-2)(t-3), \text{ zero at } t=2 \text{ and } t=3. \quad a = 12t - 30, \text{ so } a(3) = 6 \text{ m/s}^2.$$

Scoring: 1 point for the velocity zeros; 1 point for the acceleration.

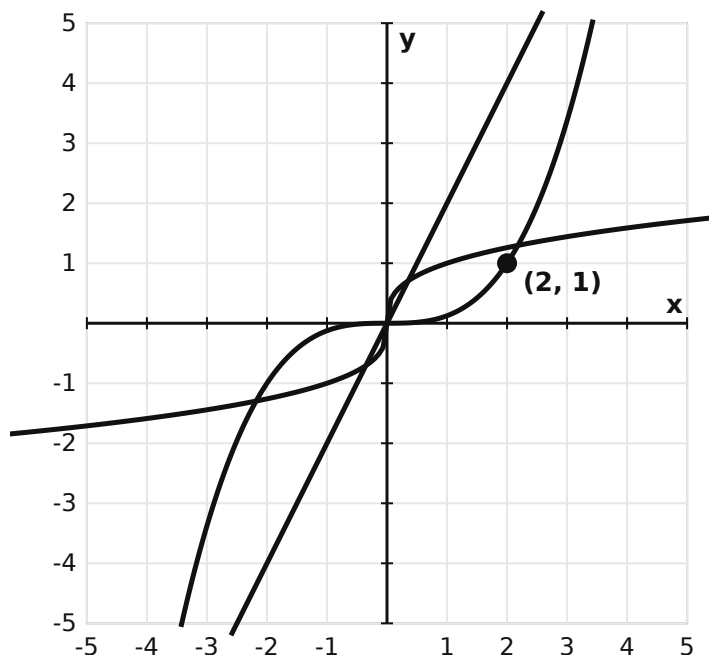
### Question 7 - Choosing rules [3 pts]

- a. Product rule:  $e^x \cos x - e^x \sin x = e^x(\cos x - \sin x)$ .
- b. Quotient rule:  $\frac{\frac{1}{2}x^2 - \ln x \cdot 2x}{x^4} = \frac{1 - 2\ln x}{x^3}$ .
- c. Chain rule:  $20(5x-1)^3$ .

Scoring: 1 point per part (rule and result).

### Question 8 - Inverse in context [3 pts]

- a.  $f(2) = 1$  and  $f'(x) = \frac{3x^2}{8}$ , so  $f'(2) = \frac{3}{2}$ .
- b.  $g'(1) = \frac{1}{f'(2)} = \frac{2}{3}$ .
- c.  $g(1) = 2$ , so  $y - 2 = \frac{2}{3}(x - 1)$ .



Graphs of  $f$  and  $g = f^{-1}$ .

Scoring: 1 point per part.

|                  |
|------------------|
| <b>Total</b>     |
| <b>20 points</b> |