

AP Calculus AB - Evaluating Limits Algebraically - Lesson

Common Core AP CALC AB 1.5, AP CALC AB 1.6, AP CALC AB 1.7, AP CALC AB 1.8 · maths-pdf.fr/en/

To evaluate a limit algebraically, first try direct substitution. If you get the indeterminate form 0/0, factor, rationalize with a conjugate, or simplify, cancel the common factor, and substitute again. The limit laws let you combine known limits, and the Squeeze Theorem handles functions trapped between two others.

AP Calculus AB · Common Core State Standards: **AP CALC AB 1.5, AP CALC AB 1.6, AP CALC AB 1.7, AP CALC AB 1.8**

Key facts

- Direct substitution works when the function is defined and continuous at the target.
- The form 0/0 means "simplify more", not "the limit is 0" and not "does not exist".
- Multiply by the conjugate to remove a square root from a limit.
- $\lim_{x \rightarrow 0} \sin(x)/x = 1$ and $\lim_{x \rightarrow 0} (1 - \cos x)/x = 0$ as x approaches 0 (radians).

What are the limit laws?

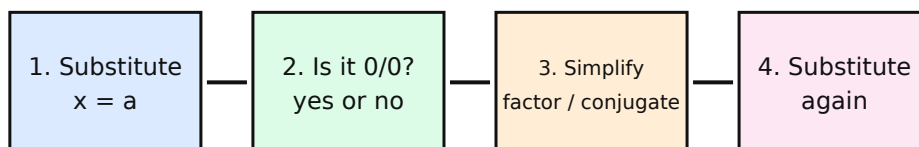
If $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = K$ both exist, limits can be combined with arithmetic.

Limit laws

- $\lim [f(x) \pm g(x)] = L \pm K$
- $\lim [c \cdot f(x)] = cL$
- $\lim [f(x) \cdot g(x)] = L \cdot K$
- $\lim \frac{f(x)}{g(x)} = \frac{L}{K}$ when $K \neq 0$
- $\lim [f(x)]^n = L^n$

For polynomials and most functions you meet in class, **direct substitution** works:

$\lim_{x \rightarrow a} f(x) = f(a)$ whenever $f(a)$ is defined.



Try direct substitution first, then simplify only if you get 0/0.

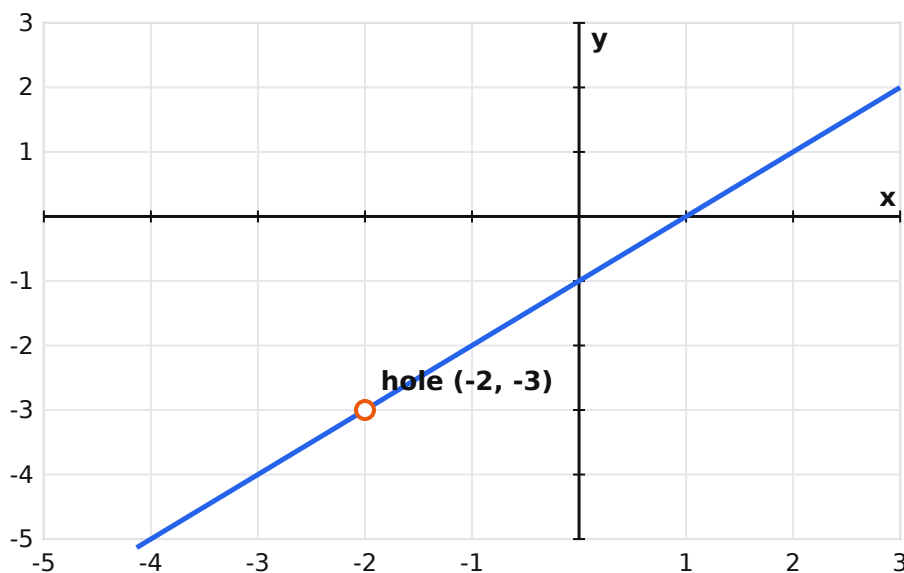
Use this quick guide to choose a method before you start any algebra.

What you see after substituting	What to do
A real number	That number is the limit.
$\frac{0}{0}$ with polynomials	Factor, cancel the common factor, substitute again.
$\frac{0}{0}$ with a square root	Multiply by the conjugate, simplify, substitute again.
A sine or cosine over x	Use the special trig limits.
A bounded oscillating factor	Try the Squeeze Theorem.

What if substitution gives 0/0?

The form $\frac{0}{0}$ is *indeterminate*: the limit may still exist. Factor the numerator and denominator, cancel the common factor, then substitute again.

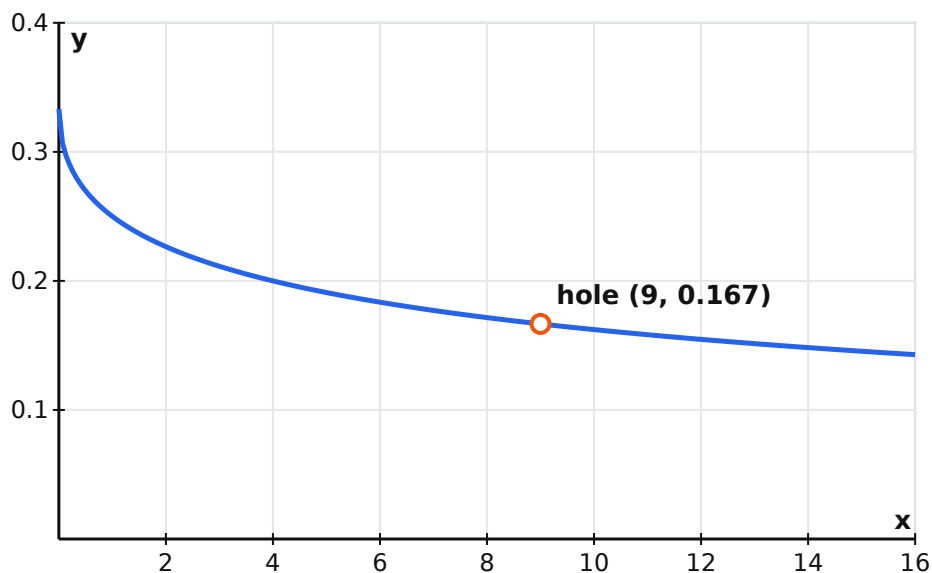
Example. $\lim_{x \rightarrow -2} \frac{x^2+x-2}{x+2} = \lim_{x \rightarrow -2} \frac{(x+2)(x-1)}{x+2} = \lim_{x \rightarrow -2} (x-1) = -3.$



The graph is the line $y = x - 1$ with a hole at $(-2, -3)$.

How do you handle square roots?

Multiply the top and bottom by the *conjugate* to remove the radical. For $\lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{x-9}$, multiply by $\frac{\sqrt{x}+3}{\sqrt{x}+3}$ to get $\frac{x-9}{(x-9)(\sqrt{x}+3)} = \frac{1}{\sqrt{x}+3}$, so the limit is $\frac{1}{6}$.

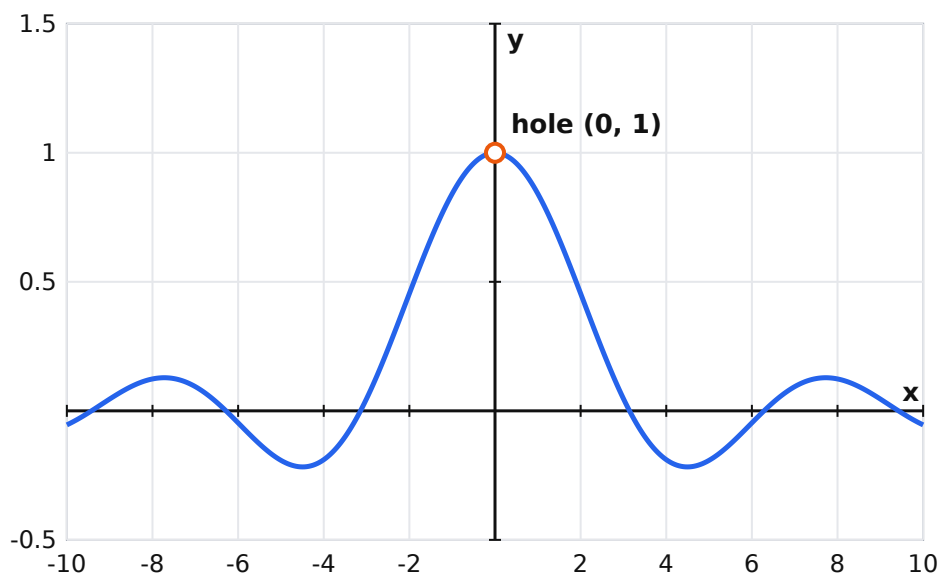


The graph of $(\sqrt{x} - 3)/(x - 9)$ has a hole at $(9, 1/6)$.

Which trig limits must you know?

Special trigonometric limits

$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ and $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$. The angle must be in radians. Rewrite $\frac{\sin 3x}{x}$ as $3 \cdot \frac{\sin 3x}{3x}$ to get 3.



The graph of $\sin x / x$ approaches 1 at $x = 0$.

What is the Squeeze Theorem?

If $g(x) \leq f(x) \leq h(x)$ near a and $\lim g(x) = \lim h(x) = L$, then $\lim f(x) = L$. Because $-x^2 \leq x^2 \sin \frac{1}{x} \leq x^2$, the middle limit at 0 is 0.

What is a clean written method?

1. Substitute the target value and look at the form of the result.
2. If the form is $\frac{0}{0}$, pick factoring, a conjugate or a special limit.
3. Simplify completely and cancel the common factor, writing "for $x \neq a$ " when it helps.
4. Substitute again and state the limit as a number.

Questions about evaluating limits algebraically

What is the first step in evaluating a limit?

Substitute the target value directly. If the result is a real number, that is the limit. If you get $0/0$, simplify the expression and try again.

Does $0/0$ mean the limit does not exist?

No. The form $0/0$ is indeterminate, so the limit could be any number or may not exist. Factor or use a conjugate to find out.

How do you use a conjugate to find a limit?

Multiply the numerator and denominator by the conjugate of the radical expression. The product of conjugates removes the square root and often creates a factor you can cancel.

Why is the limit of $\sin(x)/x$ equal to 1?

For small angles in radians, $\sin x$ is very close to x , so the ratio is very close to 1. The Squeeze Theorem proves it rigorously.

When can I use the Squeeze Theorem?

Use it when a hard function is trapped between two simpler functions that approach the same limit at the target. Then the hard function has that limit too.