

AP Calculus AB - Evaluating Limits Algebraically - Answer Key

Common Core AP CALC AB 1.5, AP CALC AB 1.6, AP CALC AB 1.7, AP CALC AB 1.8 · maths-pdf.fr/en/

This answer key gives a full worked solution for every problem in the limit algebra worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 1.5, AP CALC AB 1.6, AP CALC AB 1.7, AP CALC AB 1.8**

Exercise 1 - Direct substitution

The polynomial is continuous, so substitute: $3(2)^2 - 5(2) + 1 = 12 - 10 + 1 = 3$.

Final answer: 3.

Exercise 2 - Using the limit laws

a. $3(5) - (-2) = 17$

b. $5 \cdot (-2) = -10$

c. $\frac{5}{-2} = -\frac{5}{2}$

d. $(-2)^2 = 4$

Final answer: 17, -10, $-\frac{5}{2}$, 4.

Exercise 3 - Substitution with a root

The denominator at $x = 4$ is 2, which is not zero, so substitute directly.

$$\frac{3\sqrt{4} + 4}{4 - 2} = \frac{6 + 4}{2} = 5.$$

Final answer: 5.

Exercise 4 - Difference of squares

Substitute $x=6$: numerator 0, denominator 0. The form is $\frac{0}{0}$.

Factor: $\frac{(x-6)(x+6)}{x-6} = x+6$ for $x \neq 6$.

Substitute into the simplified expression: 12.

The graph shows a hole at (6,12). Final answer: 12.

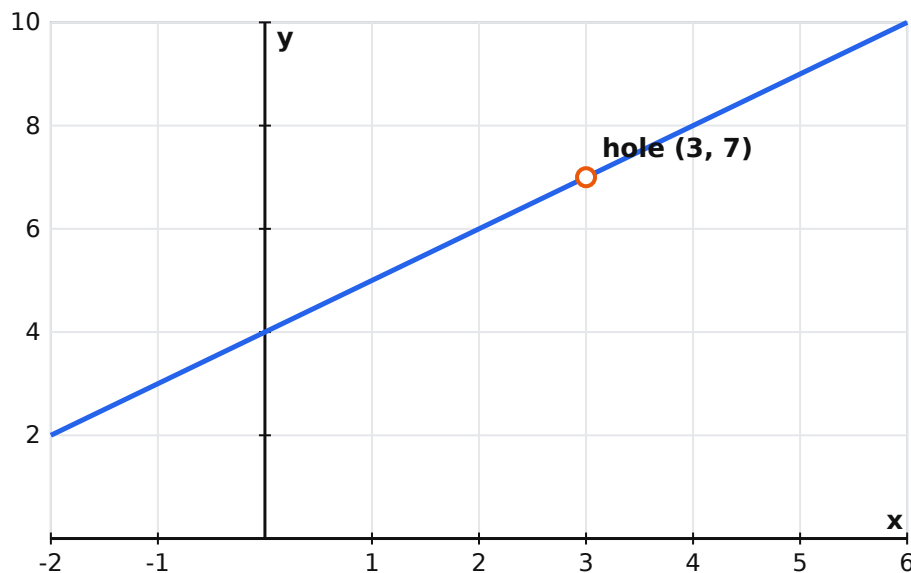
Exercise 5 - Factoring a trinomial

Substitute $x=3$: numerator 0, denominator 0. The form is $\frac{0}{0}$.

Factor: $\frac{(x-3)(x+4)}{x-3} = x+4$ for $x \neq 3$.

Substitute into the simplified expression: 7.

Final answer: 7.



Simplified graph: a hole at (3, 7).

Exercise 6 - Factoring with a leading coefficient

Substitute $x=3$: numerator 0, denominator 0. The form is $\frac{0}{0}$.

Factor: $\frac{(x-3)(2x-1)}{x-3} = 2x-1$ for $x \neq 3$.

Substitute into the simplified expression: 5.

Final answer: 5.

Exercise 7 - Difference of cubes

Substitute $x = 3$: numerator 0, denominator 0. The form is $\frac{0}{0}$.

Factor: $\frac{(x-3)(x^2+3x+9)}{x-3} = x^2+3x+9$ for $x \neq 3$.

Substitute into the simplified expression: 27.

Final answer: 27.

Exercise 8 - Repeated factor

Substitute $x = 2$: numerator 0, denominator 0. The form is $\frac{0}{0}$.

Factor: $\frac{(x-2)^2}{(x-2)(x+2)} = \frac{x-2}{x+2}$ for $x \neq 2$.

Substitute into the simplified expression: 0.

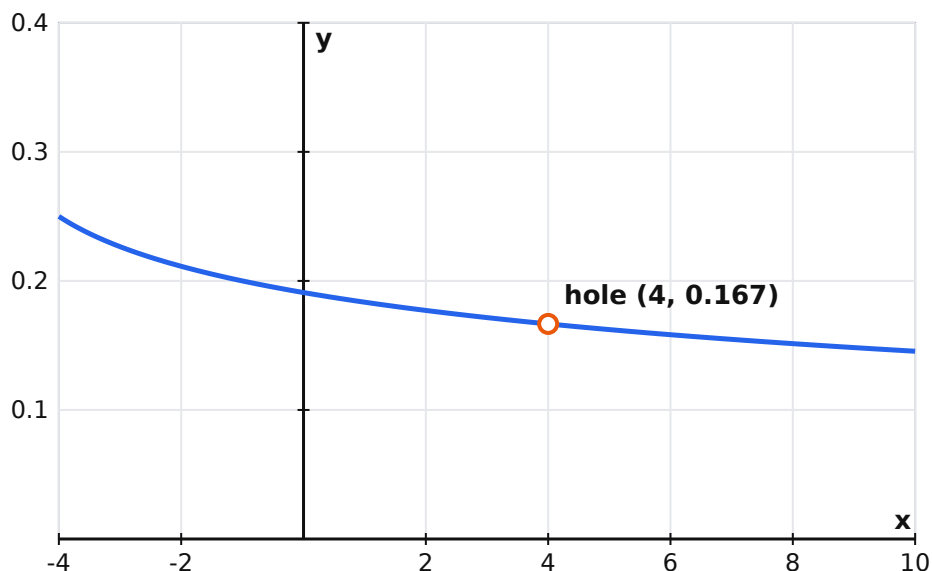
Final answer: 0.

Exercise 9 - Conjugate, example one

Direct substitution gives $\frac{0}{0}$. Multiply by the conjugate $\frac{\sqrt{x+5}+3}{\sqrt{x+5}+3}$:

$$\frac{(x+5)-9}{(x-4)(\sqrt{x+5}+3)} = \frac{x-4}{(x-4)(\sqrt{x+5}+3)} = \frac{1}{\sqrt{x+5}+3}.$$

Substitute $x = 4$: $\frac{1}{3+3} = \frac{1}{6}$. Final answer: $\frac{1}{6}$.



Hole at $(4, 1/6)$.

Exercise 10 - Conjugate, example two

Substituting gives $\frac{0}{0}$. Multiply by $\frac{\sqrt{2x+1}+3}{\sqrt{2x+1}+3}$:

$$\frac{(2x+1)-9}{(x-4)(\sqrt{2x+1}+3)} = \frac{2(x-4)}{(x-4)(\sqrt{2x+1}+3)} = \frac{2}{\sqrt{2x+1}+3}.$$

Substitute: $\frac{2}{3+3} = \frac{1}{3}$. Final answer: $\frac{1}{3}$.

Exercise 11 - Fraction with a radical

Combine the numerator: $\frac{1}{\sqrt{x}} - \frac{1}{2} = \frac{2-\sqrt{x}}{2\sqrt{x}}$.

Write $x-4 = (\sqrt{x}-2)(\sqrt{x}+2)$ and note $2-\sqrt{x} = -(\sqrt{x}-2)$: the quotient simplifies to $\frac{-1}{2\sqrt{x}(\sqrt{x}+2)}$.

Substitute $x=4$: $\frac{-1}{2 \cdot 2 \cdot 4} = -\frac{1}{16}$.

Final answer: $-\frac{1}{16}$.

Exercise 12 - Trig limit with a multiple

Rewrite: $\frac{\sin 5x}{x} = 5 \cdot \frac{\sin 5x}{5x}$. As $x \rightarrow 0$, $5x \rightarrow 0$, so $\frac{\sin 5x}{5x} \rightarrow 1$.

The limit is $5 \cdot 1 = 5$, matching the table values near 5. Final answer: 5.

Exercise 13 - Tangent over a multiple

Write $\tan 5x = \frac{\sin 5x}{\cos 5x}$, so the quotient is $\frac{5}{2} \cdot \frac{\sin 5x}{5x} \cdot \frac{1}{\cos 5x}$.

As $x \rightarrow 0$, the sine factor tends to 1 and $\cos 5x \rightarrow 1$.

Final answer: $\frac{5}{2}$.

Exercise 14 - Cosine over x sine x

Write $\frac{1 - \cos 3x}{x \sin x} = \frac{1 - \cos 3x}{(3x)^2} \cdot 9 \cdot \frac{x}{\sin x}$.

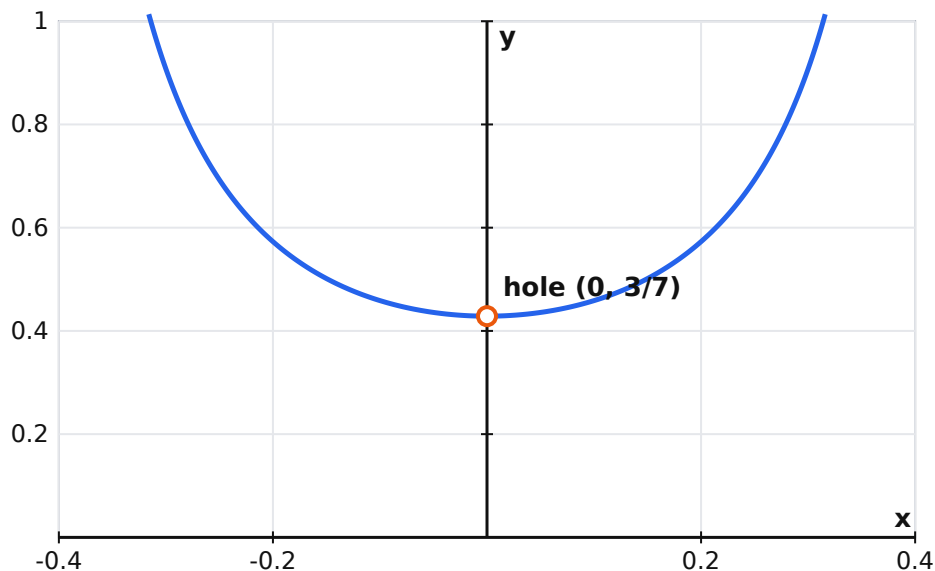
Using $\lim_{u \rightarrow 0} \frac{1 - \cos u}{u^2} = \frac{1}{2}$ (from $1 - \cos u = 2 \sin^2 \frac{u}{2}$) and $\frac{x}{\sin x} \rightarrow 1$, the limit is $\frac{1}{2} \cdot 9 \cdot 1 = \frac{9}{2}$.

Final answer: $\frac{9}{2}$.

Exercise 15 - Squeeze Theorem, cosine

For every $x \neq 0$, $-1 \leq \cos \frac{1}{x} \leq 1$. Multiply by $x^2 \geq 0$: $-x^2 \leq x^2 \cos \frac{1}{x} \leq x^2$.

Both $-x^2$ and x^2 approach 0 as $x \rightarrow 0$. By the Squeeze Theorem, the limit is 0.



The graph of $\sin 3x / \sin 7x$ approaches $3/7$.

Exercise 16 - Squeeze Theorem, given bounds

$\lim_{x \rightarrow 0} (3-x^2) = 3$ and $\lim_{x \rightarrow 0} (3+x^2) = 3$. Since g is squeezed between them, $\lim_{x \rightarrow 0} g(x) = 3$.

Exercise 17 - Finding a constant

The denominator approaches 0, so the numerator must also approach 0: $9+3k+6=0$, so $k=-5$.

Then $\frac{x^2-5x+6}{x-3} = \frac{(x-2)(x-3)}{x-3} = x-2$, and the limit is $3-2=1$.

Final answer: $k=-5$, **limit** 1.

Exercise 18 - Bakery average cost

Factor: $\frac{(n-8)(n+8)}{n-8} = n+8$ for $n \neq 8$. Substitute $n=8$: 16.

Final answer: the limit is \$16: the average cost approaches \$16 per cupcake near 8 dozen, even though the formula itself is undefined at 8.

Exercise 19 - Average speed over a short time

$s(2)=10$ and $s(2+h)=(2+h)^2+3(2+h)=4+4h+h^2+6+3h=10+7h+h^2$.

$\frac{7h+h^2}{h} = 7+h$, which approaches 7 as $h \rightarrow 0$.

Final answer: 7 ft/s.

Exercise 20 - Difference quotient

$f(3)=6$. $f(3+h)=(3+h)^2-(3+h)=9+6h+h^2-3-h=6+5h+h^2$.

$\frac{5h+h^2}{h} = 5+h$, so the limit as $h \rightarrow 0$ is 5.

Exercise 21 - Water filter efficiency

Multiply by $\frac{\sqrt{x+16}+4}{\sqrt{x+16}+4}$: $\frac{(x+16)-16}{x(\sqrt{x+16}+4)} = \frac{1}{\sqrt{x+16}+4}$.

Substitute $x=0$: $\frac{1}{4+4} = \frac{1}{8}$. Final answer: $\frac{1}{8}$.

Exercise 22 - Finding the error

The special limit needs the same expression inside the sine and in the denominator: $\frac{\sin u}{u} \rightarrow 1$. Here the sine has $2x$ but the denominator has x .

Rewrite $\frac{\sin 2x}{x} = 2 \cdot \frac{\sin 2x}{2x}$, which approaches $2 \cdot 1 = 2$.

Final answer: Liam's result is wrong; the limit is 2.