

AP Calculus AB - Evaluating Limits Algebraically - Worksheet

Common Core AP CALC AB 1.5, AP CALC AB 1.6, AP CALC AB 1.7, AP CALC AB 1.8 · maths-pdf.fr/en/

Name: _____ Class: _____ Date: ____/____/____

These 22 problems practice evaluating limits with substitution, factoring, conjugates, special trig limits and the Squeeze Theorem (AP Calculus AB, topics 1.5, 1.6, 1.7, 1.8). The early problems build the basic skill, and each one uses a different function or situation so that no two feel alike.

Later problems use difference quotients, finding an unknown constant, and real situations such as average cost per cupcake and scooter speed. Work in pencil, show every step, and check the answer key only after a full attempt.

AP Calculus AB · Common Core State Standards: **AP CALC AB 1.5, AP CALC AB 1.6, AP CALC AB 1.7, AP CALC AB 1.8**

Exercise 1 - Direct substitution

Find $\lim_{x \rightarrow 2} 3x^2 - 5x + 1$.

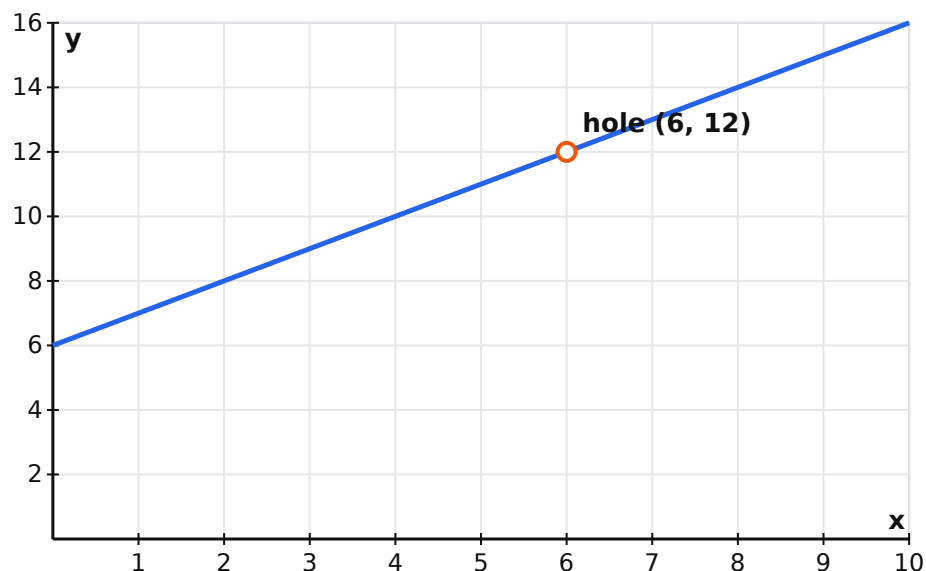
Exercise 2 - Using the limit laws

Suppose $\lim_{x \rightarrow 4} f(x) = 5$ and $\lim_{x \rightarrow 4} g(x) = -2$. Find $\lim_{x \rightarrow 4} [3f(x) - g(x)]$, $\lim_{x \rightarrow 4} [f(x) \cdot g(x)]$, $\lim_{x \rightarrow 4} \frac{f(x)}{g(x)}$ and $\lim_{x \rightarrow 4} [g(x)]^2$.

Exercise 3 - Substitution with a root

Find $\lim_{x \rightarrow 4} \frac{3\sqrt{x} + x}{x - 2}$.

Exercise 4 - Difference of squares



The graph of $(x^2 - 36)/(x - 6)$.

Find $\lim_{x \rightarrow 6} \frac{x^2 - 36}{x - 6}$. Use the graph to check your answer.

Exercise 5 - Factoring a trinomial

Find $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x - 3}$.

Exercise 6 - Factoring with a leading coefficient

Find $\lim_{x \rightarrow 3} \frac{2x^2 - 7x + 3}{x - 3}$.

Exercise 7 - Difference of cubes

Find $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x - 3}$.

Exercise 8 - Repeated factor

Find $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x^2 - 4}$.

Exercise 9 - Conjugate, example one

Find $\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x - 4}$.

Exercise 10 - Conjugate, example two

Find $\lim_{x \rightarrow 4} \frac{\sqrt{2x+1} - 3}{x - 4}$.

Exercise 11 - Fraction with a radical

Find $\lim_{x \rightarrow 4} \frac{\frac{1}{\sqrt{x}} - \frac{1}{2}}{x - 4}$.

Exercise 12 - Trig limit with a multiple

x	-0.1	-0.01	0.01	0.1
$\sin(5x)/x$	4.79426	4.99792	4.99792	4.79426

Values of $\sin(5x)/x$ near 0.

The table suggests the value of $\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$. Find the exact limit algebraically.

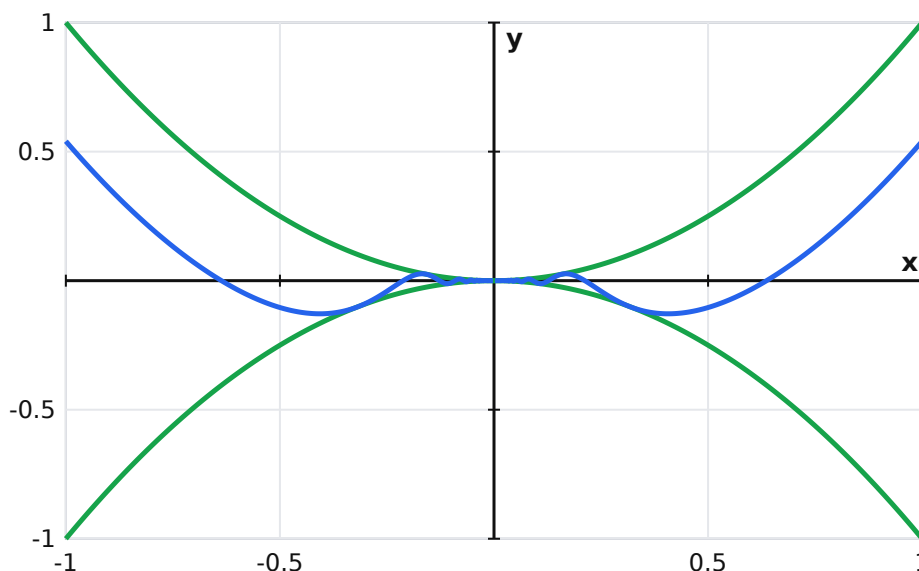
Exercise 13 - Tangent over a multiple

Find $\lim_{x \rightarrow 0} \frac{\tan 5x}{2x}$.

Exercise 14 - Cosine over x sine x

Find $\lim_{x \rightarrow 0} \frac{1 - \cos 3x}{x \sin x}$.

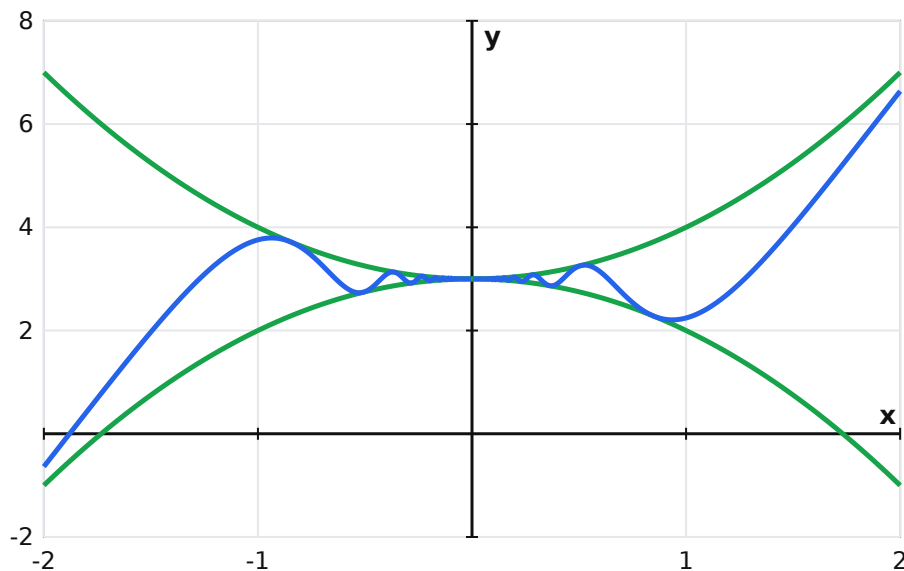
Exercise 15 - Squeeze Theorem, cosine



The blue curve is squeezed between $y = x^2$ and $y = -x^2$.

Use the Squeeze Theorem to find $\lim_{x \rightarrow 0} x^2 \cos \frac{1}{x}$.

Exercise 16 - Squeeze Theorem, given bounds



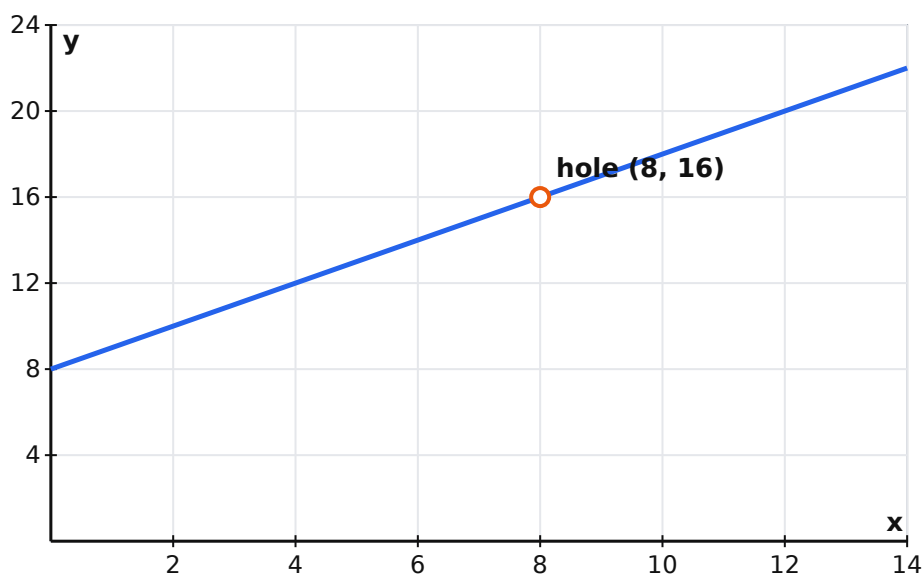
g is trapped between $3 - x^2$ and $3 + x^2$.

A function g satisfies $3 - x^2 \leq g(x) \leq 3 + x^2$ for all x . Find $\lim_{x \rightarrow 0} g(x)$.

Exercise 17 - Finding a constant

Find the value of k for which $\lim_{x \rightarrow 3} \frac{x^2 + kx + 6}{x - 3}$ exists, and then find the limit.

Exercise 18 - Bakery average cost



Average cost per cupcake: a hole at $n = 8$.

Priya models the average cost in dollars of a cupcake as $A(n) = \frac{n^2 - 64}{n - 8}$ for a batch of n dozen.

Find $\lim_{n \rightarrow 8} A(n)$ and say what it means.

Exercise 19 - Average speed over a short time

A scooter's position is $s(t) = t^2 + 3t$ feet after t seconds. Simplify $\frac{s(2+h) - s(2)}{h}$ and find its limit as $h \rightarrow 0$.

Exercise 20 - Difference quotient

For $f(x) = x^2 - x$, find $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$.

Exercise 21 - Water filter efficiency

The efficiency of a filter is $E(x) = \frac{\sqrt{x+16} - 4}{x}$ at a flow setting x . Find $\lim_{x \rightarrow 0} E(x)$.

Exercise 22 - Finding the error

Liam writes $\lim_{x \rightarrow 0} \frac{\sin 2x}{x} = 1$ because "sine over x is 1". Explain the mistake and give the correct limit.