

AP Calculus AB - Introducing Limits - Lesson

Common Core AP CALC AB 1.1, AP CALC AB 1.2, AP CALC AB 1.3, AP CALC AB 1.4 · maths-pdf.fr/en/

A limit is the value that a function approaches as its input gets closer and closer to a target number, without needing to reach it. Written as $\lim f(x) = L$, it tells you where the outputs are heading from both sides of the target. A limit can exist even when the function is undefined at that point.

AP Calculus AB · Common Core State Standards: **AP CALC AB 1.1, AP CALC AB 1.2, AP CALC AB 1.3, AP CALC AB 1.4**

Key facts

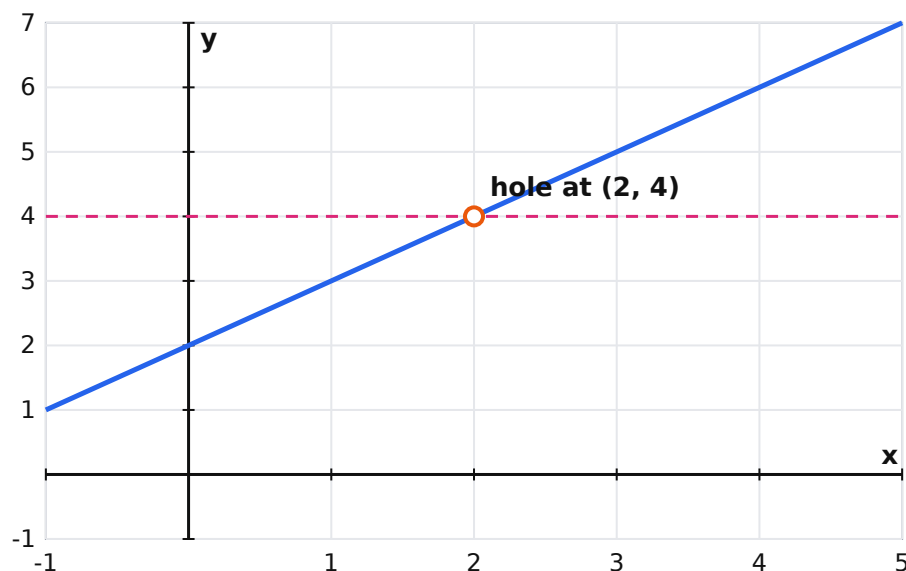
- A limit describes nearby behavior, not the value at the point itself.
- The two-sided limit exists only if the left-hand and right-hand limits are equal.
- Tables and graphs give estimates; algebra gives exact values.
- A limit fails to exist because of a jump, unbounded growth or oscillation.

What is a limit?

A limit describes where the outputs of a function head as the input gets closer and closer to a target number. It never asks what happens exactly at the target, only what happens nearby. This idea is the foundation of every derivative and integral in calculus.

Limit notation

$\lim_{x \rightarrow a} f(x) = L$ means that $f(x)$ gets arbitrarily close to L as x gets close to a from both sides, with $x \neq a$.



Near $x = 2$ the outputs approach 4, even though the graph has a hole at $x = 2$.

How do you estimate a limit from a table?

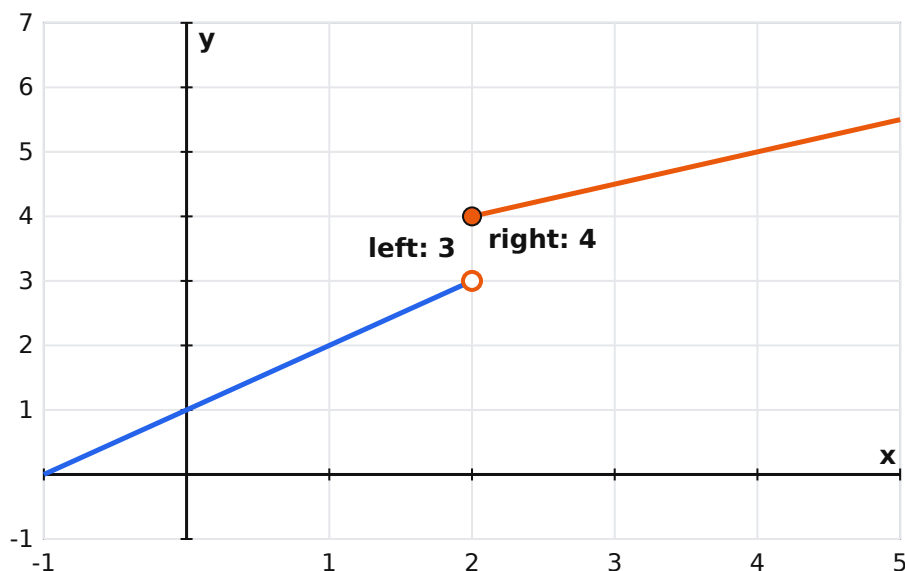
Choose inputs just below and just above the target, such as 1.9, 1.99, 1.999 and 2.001, 2.01, 2.1. If the outputs settle toward one number from both sides, that number is the limit.

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	3.9	3.99	3.999	4.001	4.01	4.1

Outputs squeeze toward 4 from both sides of $x = 2$.

What are one-sided limits?

The left-hand limit $\lim_{x \rightarrow a^-} f(x)$ uses only inputs less than a . The right-hand limit $\lim_{x \rightarrow a^+} f(x)$ uses only inputs greater than a . The two-sided limit exists only when both one-sided limits exist and are equal.



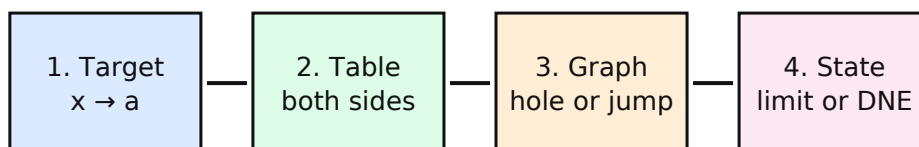
The left-hand limit is 3 and the right-hand limit is 4, so the two-sided limit does not exist.

When does a limit fail to exist?

- **Jump:** the left and right limits are different numbers.
- **Unbounded behavior:** the outputs grow without bound, as with $\frac{1}{x^2}$ near $x=0$.
- **Oscillation:** the outputs swing between values forever, as with $\sin\left(\frac{1}{x}\right)$ near $x=0$.

Remember that $f(a)$ and $\lim_{x \rightarrow a} f(x)$ can be different, or $f(a)$ may not exist at all.

What is a reliable method?



A reliable routine for estimating a limit.

1. Name the target value a and the function.
2. Build a table with inputs on both sides of a .
3. Check the graph for a hole, jump or asymptote.
4. State the limit, or explain why it does not exist.

Example. Estimate $\lim_{x \rightarrow 3} \frac{x^2-9}{x-3}$. At $x=2.99$ the output is 5.99, and at $x=3.01$ it is 6.01. The outputs approach 6 from both sides.

Questions about introducing limits

What does a limit mean in calculus?

A limit is the number a function approaches as the input approaches a target value. It describes behavior near the target, so the function does not need to be defined there.

Can a limit exist if the function is undefined at that point?

Yes. For $f(x) = (x^2 - 4)/(x - 2)$, the function is undefined at $x = 2$, but the outputs approach 4 from both sides, so the limit is 4.

What is a one-sided limit?

A one-sided limit looks at inputs from only one side of the target. The notation $x \rightarrow a^+$ means from the right, and $x \rightarrow a^-$ means from the left.

When does a limit not exist?

A limit does not exist when the left and right limits differ, when the outputs grow without bound, or when they oscillate without settling on a single value.

How does a limit connect to a derivative?

The derivative is defined as the limit of an average rate of change as the interval shrinks to zero. Limits turn average rates into instantaneous rates.