

AP Calculus AB - Introducing Limits - Answer Key

Common Core AP CALC AB 1.1, AP CALC AB 1.2, AP CALC AB 1.3, AP CALC AB 1.4 · maths-pdf.fr/en/

This answer key gives a full worked solution for every problem in the limits worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 1.1, AP CALC AB 1.2, AP CALC AB 1.3, AP CALC AB 1.4**

Exercise 1 - Reading limit notation

The notation says that as x gets closer and closer to 5 from both sides, the outputs $f(x)$ get closer and closer to 12.

Final answer: **the limit of $f(x)$ as x approaches 5 is 12.**

Exercise 2 - Table for a linear function

Add 4 to each input: 6.9, 6.99, 6.999, 7.001, 7.01, 7.1.

The outputs approach 7 from both sides. Final answer: $\lim_{x \rightarrow 7} (x+4) = 7$.

Exercise 3 - Table with an undefined point

Evaluate each input: 5.9, 5.99, 5.999, 6.001, 6.01, 6.1.

Although $g(3)$ is undefined, the outputs approach 6. Final answer: 6.

Exercise 4 - Limit versus function value

Follow the line from the left and from the right toward $x=2$. Both sides approach height 3.

The filled point is at (2,1), so the actual value is $f(2)=1$.

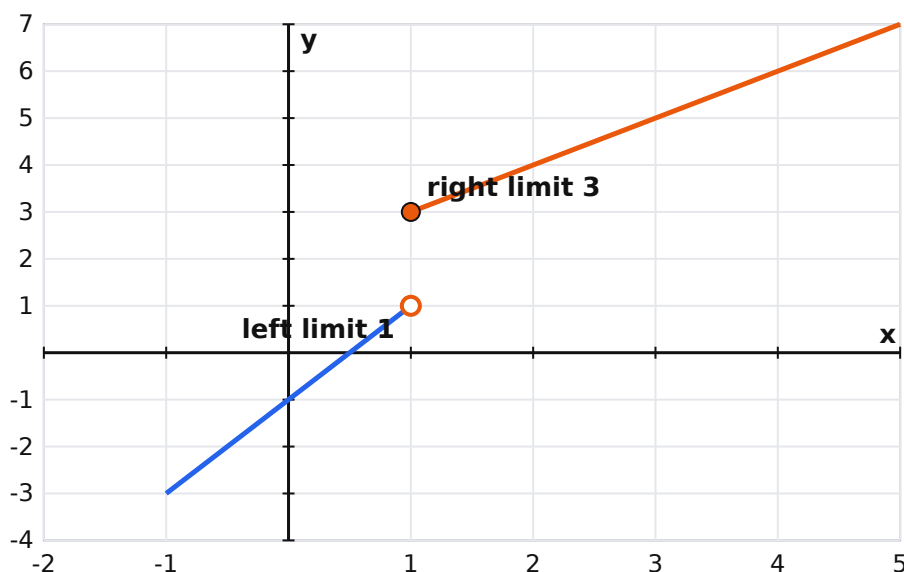
Final answer: $\lim_{x \rightarrow 2} f(x) = 3$ **and** $f(2) = 1$.

Exercise 5 - One-sided limits from a graph

From the left, the graph follows $y = 2x - 1$ and reaches height $2(1) - 1 = 1$. So the left-hand limit is 1.

From the right, it follows $y = x + 2$ and starts at $1 + 2 = 3$. So the right-hand limit is 3.

The one-sided limits differ, so $\lim_{x \rightarrow 1} f(x)$ **does not exist**.



The two one-sided limits are 1 and 3.

Exercise 6 - Limit at a smooth point

The curve is unbroken at $x = 2$, so the outputs approach the point on the graph.

Compute $f(2) = \frac{4}{2} - 2 = 0$. Final answer: 0.

Exercise 7 - Absolute value quotient

For negative inputs, $|x| = -x$, so the quotient is -1 . For positive inputs it is 1. Table outputs: $-1, -1, -1, 1, 1, 1$.

The left-hand limit is -1 and the right-hand limit is 1. Final answer: **the limit does not exist** because the one-sided limits differ.

Exercise 8 - Unbounded outputs

At $x = \pm 0.5$ the output is 4; at $x = \pm 0.1$ it is 100; at $x = \pm 0.01$ it is 10,000. The outputs grow without bound from both sides.

Final answer: the outputs increase without bound, so **the limit does not exist as a real number**. In symbols, the limit is ∞ (a vertical asymptote at $x = 0$).

Exercise 9 - Sine over x

The outputs are 0.99833, 0.99998, 0.99998, 0.99833.

They approach 1 from both sides. Final answer: 1.

Exercise 10 - Square root difference

The outputs are 0.25158, 0.25016, 0.25002, 0.24998, 0.24984, 0.24846.

They approach 0.25 from both sides, so the limit is $\frac{1}{4}$.

Exercise 11 - Average rate of change

$$\frac{f(3) - f(1)}{3 - 1} = \frac{9 - 1}{2} = 4.$$

$$\frac{f(1.1) - f(1)}{0.1} = \frac{1.21 - 1}{0.1} = 2.1. \text{ And } \frac{1.0201 - 1}{0.01} = 2.01.$$

The averages approach 2. Final answer: 4, 2.1, 2.01; **instantaneous rate about 2**.

Exercise 12 - Limit from a table

As x approaches 5 from the left, $g(x)$ is 2.9, 2.99, 2.999. From the right it is 3.1, 3.01, 3.001.

Both sides approach 3. The table lists $g(5) = 1$, a different number.

Final answer: $\lim_{x \rightarrow 5} g(x) = 3$ **and** $g(5) = 1$.

Exercise 13 - Piecewise one-sided limits

Left: use x^2 , so $\lim_{x \rightarrow 1^-} f(x) = 1^2 = 1$.

Right: use $2x+1$, so $\lim_{x \rightarrow 1^+} f(x) = 2(1)+1 = 3$.

Final answer: left 1, right 3, and **the two-sided limit does not exist**.

Exercise 14 - Making a limit exist

Left-hand limit: $3(2)+1=7$. Right-hand limit: $2^2+k=4+k$.

Set them equal: $4+k=7$, so $k=3$.

Final answer: $k=3$.

Exercise 15 - Square root near the edge

$\sqrt{0.1} \approx 0.3162$, $\sqrt{0.01} = 0.1$, $\sqrt{0.001} \approx 0.0316$. The outputs approach 0, so the right-hand limit is 0.

For $x < 1$ the expression $x-1$ is negative, so $\sqrt{x-1}$ is not a real number. There are no inputs on the left to test.

Exercise 16 - Parking garage fee

Just below 2 hours (for example $t=1.99$), the fee is \\$6. So $\lim_{t \rightarrow 2^-} C(t) = 6$.

Just above 2 hours (for example $t=2.01$), the fee is \\$9. So $\lim_{t \rightarrow 2^+} C(t) = 9$.

Final answer: left \\$6, right \\$9, and **the two-sided limit does not exist**.

Exercise 17 - Speed of a skateboarder

$\frac{s(3+h)-s(3)}{h}$ with $s(3)=9$: $\frac{16-9}{1}=7$, $\frac{9.61-9}{0.1}=6.1$, $\frac{9.0601-9}{0.01}=6.01$, $\frac{9.006001-9}{0.001}=6.001$.

The averages approach 6. Final answer: about 6 m/s at $t=3$.

Exercise 18 - Cooling coffee

$\frac{150-180}{2} = -15$, $\frac{128-150}{2} = -11$, $\frac{110-128}{2} = -9$, $\frac{96-110}{2} = -7$ degrees per minute.

The coffee cools more slowly as time passes. Final answer: $-15, -11, -9, -7$ $^{\circ}\text{F}/\text{min}$; near $t=8$ the rate is about -7 $^{\circ}\text{F}$ per minute or a little slower.

Exercise 19 - Filling a water tank

$$V(5) = 100. \text{ Then } \frac{V(5.1) - 100}{0.1} = \frac{104.04 - 100}{0.1} = 40.4.$$

$$\frac{100.4004 - 100}{0.01} = 40.04 \text{ and } \frac{100.040004 - 100}{0.001} = 40.004.$$

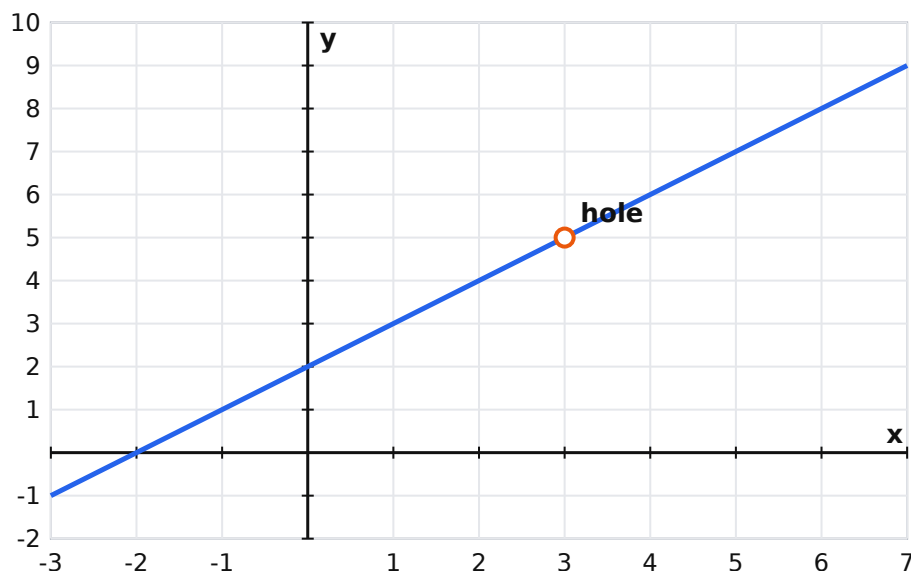
The averages approach 40. Final answer: 40 gallons per minute at $t=5$.

Exercise 20 - Hole in a rational graph

(a) At $x=3$ the denominator is 0 and the numerator is $9-3-6=0$, so $g(3)=\frac{0}{0}$ is undefined.

(b) The outputs are 4.9, 4.99, 5.01, 5.1, approaching 5 from both sides.

(c) The graph follows the line $y = x+2$ with a hole at $(3,5)$.



The hole in the graph of g sits at $(3, 5)$.

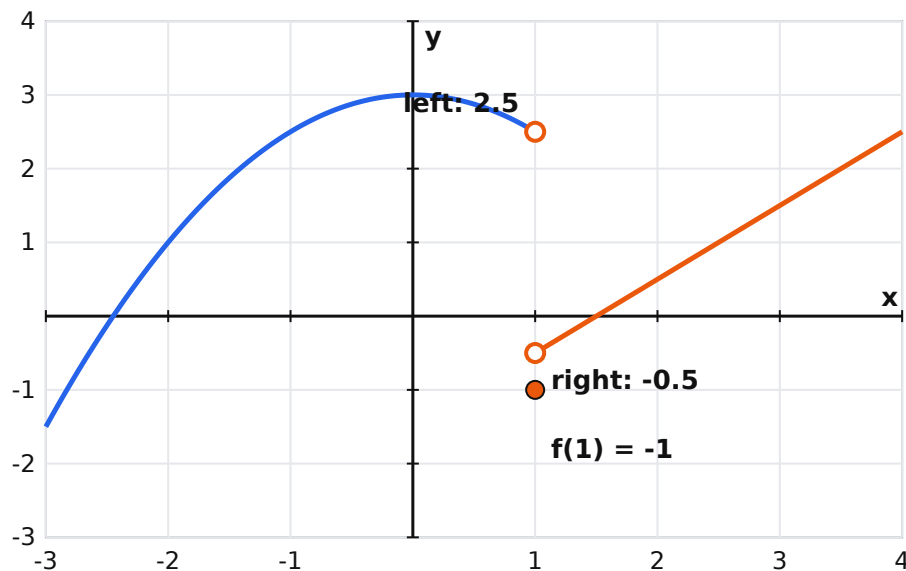
Final answer: $\lim_{x \rightarrow 3} g(x) = 5$ and a hole at $(3, 5)$.

Exercise 21 - Several limits from one graph

a. The parabola reaches height $-0.5(1)^2 + 3 = 2.5$, so the left-hand limit is 2.5.

b. The line starts at $1 - 1.5 = -0.5$ as $x \rightarrow 1^+$, so the right-hand limit is -0.5 .

- c. The filled point is at $(1, -1)$, so $f(1) = -1$.
- d. The one-sided limits differ, so **the limit does not exist**.
- e. The graph is unbroken at $x=3$ with $f(3) = 3 - 1.5 = 1.5$, so the limit is 1.5.



Labels for the one-sided limits and $f(1)$.

Exercise 22 - Does the limit equal the value?

For $x \neq 4$, $h(x) = x + 4$ (since $x^2 - 16 = (x - 4)(x + 4)$). The table gives 7.9, 7.99, 8.01, 8.1, approaching 8.

But $h(4) = 2$. Since $\lim_{x \rightarrow 4} h(x) = 8 \neq 2 = h(4)$, the limit is not the function value.

Final answer: **Jamal is wrong**; the limit is 8 while $h(4) = 2$.