

AP Calculus AB - Introducing Limits - Worksheet

Common Core AP CALC AB 1.1, AP CALC AB 1.2, AP CALC AB 1.3, AP CALC AB 1.4 · maths-pdf.fr/en/

Name: _____ Class: _____ Date: ____/____/____

These 22 problems practice estimating limits from tables and graphs, one-sided limits and the cases where a limit does not exist (AP Calculus AB, topics 1.1, 1.2, 1.3, 1.4). The early problems build the basic skill, and each one uses a different function or situation so that no two feel alike.

Later problems use piecewise functions, rates of change from tables, and real situations such as parking fees and a cooling coffee. Work in pencil, show every step, and check the answer key only after a full attempt.

AP Calculus AB · Common Core State Standards: **AP CALC AB 1.1, AP CALC AB 1.2, AP CALC AB 1.3, AP CALC AB 1.4**

Exercise 1 - Reading limit notation

Write $\lim_{x \rightarrow 5} f(x) = 12$ in words.

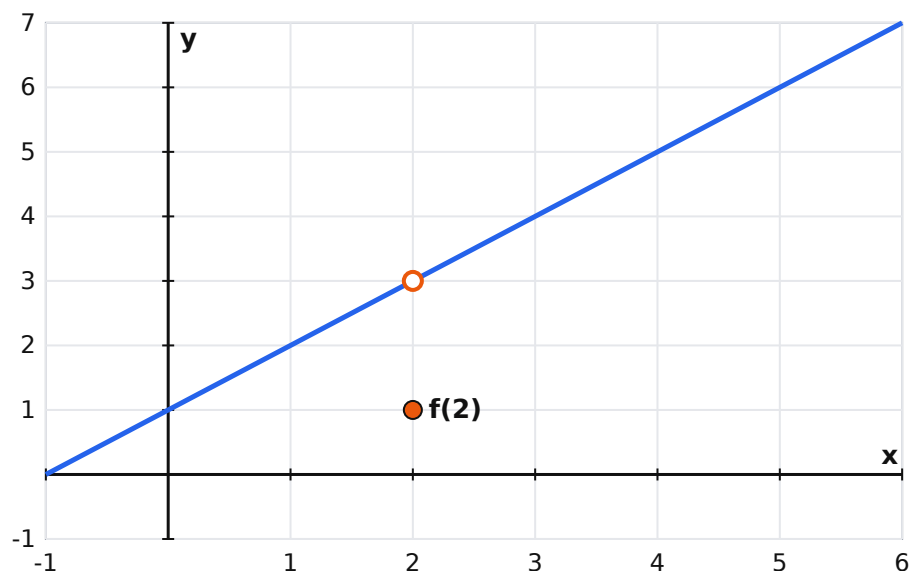
Exercise 2 - Table for a linear function

Complete a table for $f(x) = x + 4$ at 2.9, 2.99, 2.999, 3.001, 3.01, 3.1, then estimate $\lim_{x \rightarrow 3} f(x)$.

Exercise 3 - Table with an undefined point

Make a table for $g(x) = \frac{x^2 - 9}{x - 3}$ using 2.9, 2.99, 2.999, 3.001, 3.01, 3.1 and estimate $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$.

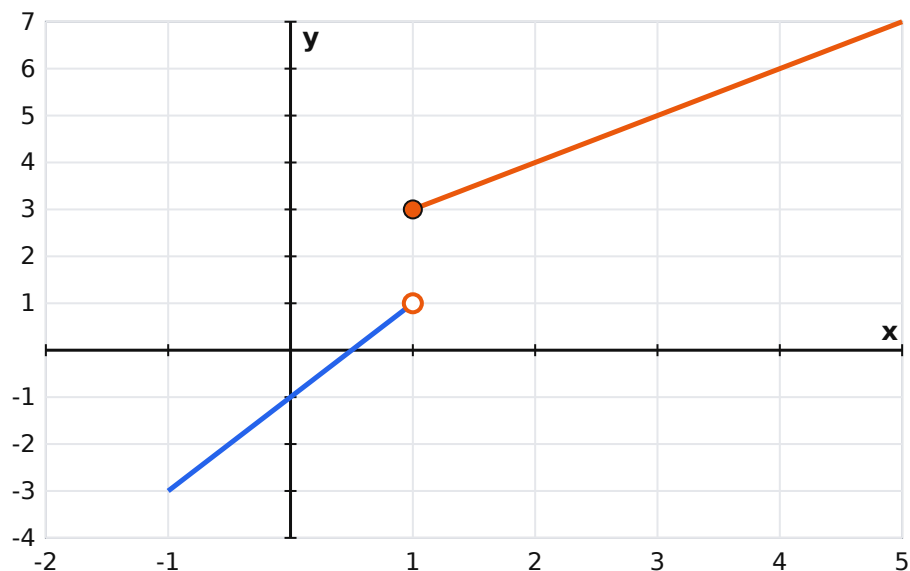
Exercise 4 - Limit versus function value



The graph of f has an open circle at $(2, 3)$ and a filled point at $(2, 1)$.

Use the graph of f to find $\lim_{x \rightarrow 2} f(x)$ and $f(2)$.

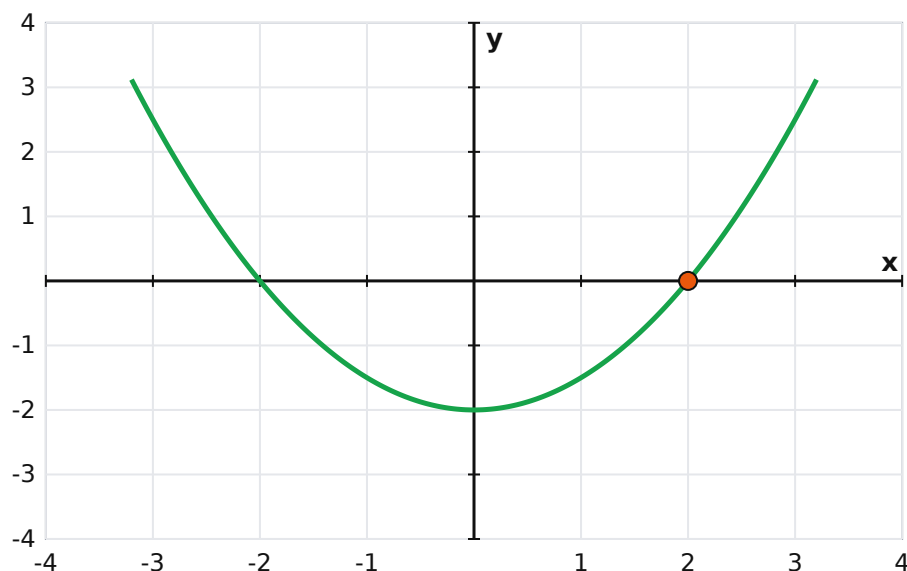
Exercise 5 - One-sided limits from a graph



A jump at $x = 1$: open circle at $(1, 1)$, filled point at $(1, 3)$.

Use the graph to find $\lim_{x \rightarrow 1^-} f(x)$, $\lim_{x \rightarrow 1^+} f(x)$ and $\lim_{x \rightarrow 1} f(x)$.

Exercise 6 - Limit at a smooth point



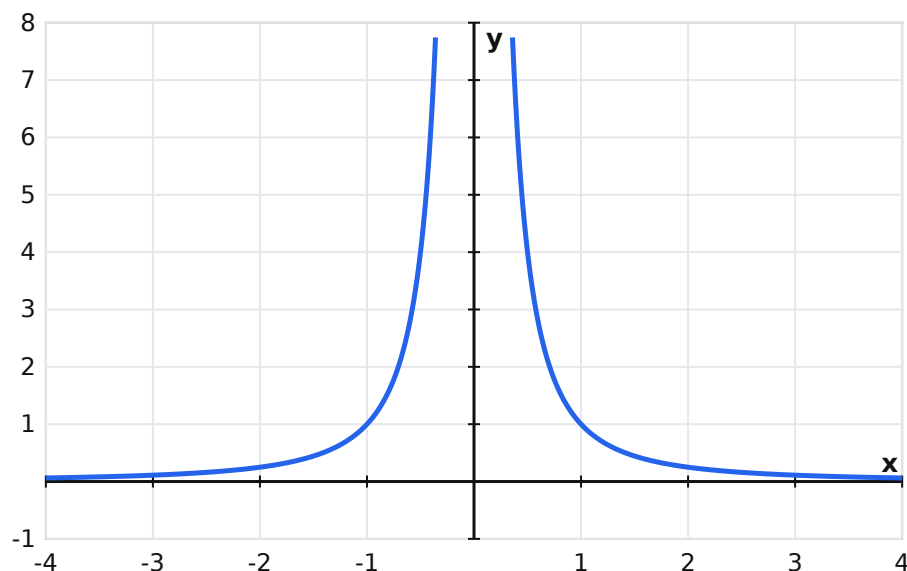
The graph of $f(x) = x^2/2 - 2$ is a smooth parabola.

The graph shows $f(x) = \frac{x^2}{2} - 2$. Find $\lim_{x \rightarrow 2} f(x)$.

Exercise 7 - Absolute value quotient

Estimate $\lim_{x \rightarrow 0} \frac{|x|}{x}$ using inputs $-0.1, -0.01, -0.001, 0.001, 0.01, 0.1$. Does the limit exist?

Exercise 8 - Unbounded outputs



Both branches of $1/x^2$ climb without bound near $x = 0$.

The graph shows $f(x) = \frac{1}{x^2}$. Describe the behavior as $x \rightarrow 0$ and state whether the limit exists as a real number.

Exercise 9 - Sine over x

With the calculator in radian mode, evaluate $\frac{\sin x}{x}$ at $-0.1, -0.01, 0.01, 0.1$ and estimate $\lim_{x \rightarrow 0} \frac{\sin x}{x}$.

Exercise 10 - Square root difference

Estimate $\lim_{x \rightarrow 0} \frac{\sqrt{x+4}-2}{x}$ by evaluating the quotient at $-0.1, -0.01, -0.001, 0.001, 0.01, 0.1$.

Exercise 11 - Average rate of change

For $f(x) = x^2$, find the average rate of change on $[1, 3]$, $[1, 1.1]$ and $[1, 1.01]$. Then estimate the instantaneous rate of change at $x = 1$.

Exercise 12 - Limit from a table

x	4.9	4.99	4.999	5	5.001	5.01	5.1
g(x)	2.9	2.99	2.999	1	3.001	3.01	3.1

Values of g near $x = 5$.

Use the table to estimate $\lim_{x \rightarrow 5} g(x)$ and to state $g(5)$.

Exercise 13 - Piecewise one-sided limits

Let $f(x) = x^2$ for $x < 1$ and $f(x) = 2x + 1$ for $x \geq 1$. Find the left-hand limit, the right-hand limit and the limit at $x = 1$.

Exercise 14 - Making a limit exist

Let $f(x) = 3x + 1$ for $x < 2$ and $f(x) = x^2 + k$ for $x \geq 2$. Find k so that $\lim_{x \rightarrow 2} f(x)$ exists.

Exercise 15 - Square root near the edge

Estimate $\lim_{x \rightarrow 1^+} \sqrt{x-1}$ using $x = 1.1, 1.01, 1.001$. Why does the left-hand limit at $x = 1$ not make sense for this function?

Exercise 16 - Parking garage fee

A garage charges \$3 for each hour or part of an hour. The fee in dollars for parking t hours is $C(t)$, with $C(t) = 6$ for $1 < t \leq 2$ and $C(t) = 9$ for $2 < t \leq 3$. Find the left-hand and right-hand limits of $C(t)$ as $t \rightarrow 2$ and decide whether the two-sided limit exists.

Exercise 17 - Speed of a skateboarder

Maya rolls down a ramp, and her distance from the top after t seconds is $s(t) = t^2$ meters. Find her average speed over $[3, 4]$, $[3, 3.1]$, $[3, 3.01]$ and $[3, 3.001]$, then estimate her speed at $t = 3$ seconds.

Exercise 18 - Cooling coffee

The temperature of a coffee in $^{\circ}\text{F}$ is T at t minutes: $T(0) = 180$, $T(2) = 150$, $T(4) = 128$, $T(6) = 110$, $T(8) = 96$. Find the average rate of change on each 2-minute interval. What do the averages suggest about the rate near $t = 8$?

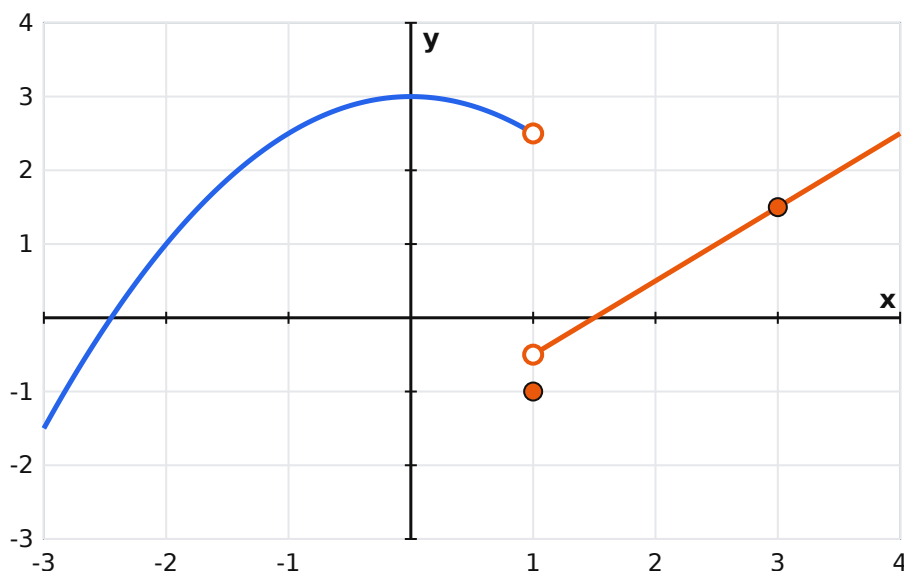
Exercise 19 - Filling a water tank

A tank holds $V(t) = 4t^2$ gallons after t minutes. Find the average rate of change over $[5, 5.1]$, $[5, 5.01]$ and $[5, 5.001]$, and estimate the instantaneous rate at $t = 5$.

Exercise 20 - Hole in a rational graph

Let $g(x) = \frac{x^2 - x - 6}{x - 3}$. (a) Explain why $g(3)$ is undefined. (b) Estimate $\lim_{x \rightarrow 3} g(x)$ with a table at 2.9, 2.99, 3.01, 3.1. (c) Describe the graph at $x = 3$.

Exercise 21 - Several limits from one graph



The graph of f has a jump at $x = 1$.

Use the graph of f to find: (a) $\lim_{x \rightarrow 1^-} f(x)$; (b) $\lim_{x \rightarrow 1^+} f(x)$; (c) $f(1)$; (d) $\lim_{x \rightarrow 1} f(x)$; (e) $\lim_{x \rightarrow 3} f(x)$.

Exercise 22 - Does the limit equal the value?

Jamal says, "The limit of a function at $x=4$ is always $f(4)$." Use $h(x) = \frac{x^2-16}{x-4}$ for $x \neq 4$ with $h(4)=2$ to show that he is wrong. Compute the limit using a table with 3.9, 3.99, 4.01, 4.1.