

AP Calculus AB - Linearization and L'Hopital's Rule - Answer Key

Common Core AP CALC AB 4.5, AP CALC AB 4.6, AP CALC AB 4.7 · maths-pdf.fr/en/

This answer key gives a full worked solution for every problem in the linearization worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 4.5, AP CALC AB 4.6, AP CALC AB 4.7**

Exercise 1 - Linearization of x squared

$$f(3)=9 \text{ and } f'(3)=6, \text{ so } L(x)=9+6(x-3).$$

$$L(3.1)=9+6(0.1)=9.6. \text{ The exact value is } 9.61, \text{ so the error is } 0.01.$$

$$\text{Final answer: } L(x)=9+6(x-3); (3.1)^2 \approx 9.6.$$

Exercise 2 - Estimating a square root

$$f(25)=5, f'(x)=\frac{1}{2\sqrt{x}}, \text{ so } f'(25)=\frac{1}{10}.$$

$$L(26)=5+\frac{1}{10}(1)=5.1. \text{ The calculator gives } 5.0990.$$

$$\text{Final answer: } \sqrt{26} \approx 5.1.$$

Exercise 3 - Estimating a cube root

$$f(27)=3 \text{ and } f'(x)=\frac{1}{3}x^{-2/3}, \text{ so } f'(27)=\frac{1}{3 \cdot 9}=\frac{1}{27}.$$

$$L(28)=3+\frac{1}{27} \approx 3.037.$$

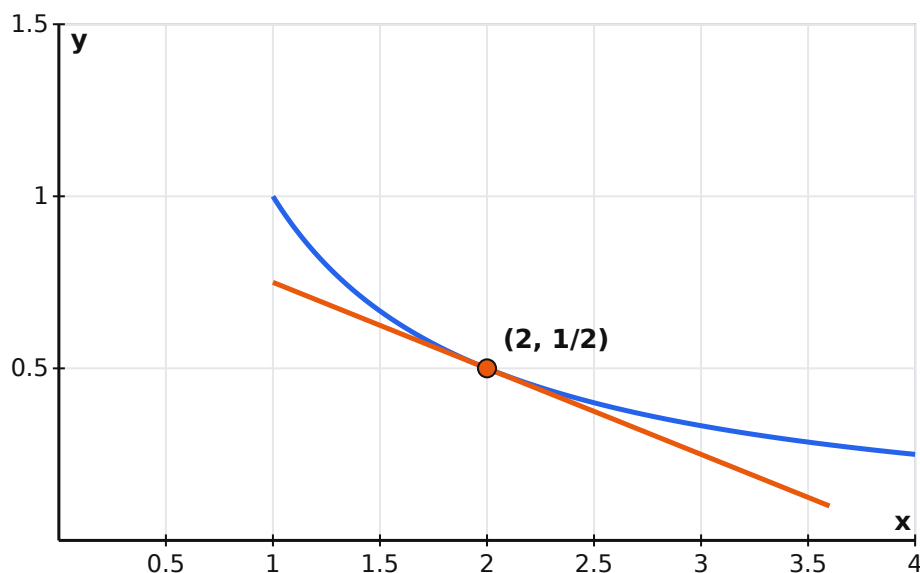
$$\text{Final answer: } \sqrt[3]{28} \approx 3.037.$$

Exercise 4 - Estimating a reciprocal

$$f(2) = \frac{1}{2} \text{ and } f'(2) = -\frac{1}{4}.$$

$$L(2.1) = \frac{1}{2} - \frac{1}{4}(0.1) = 0.475. \text{ The exact value is } 0.47619.$$

Final answer: 0.475.



The tangent line $L(x) = 1/2 - (x - 2)/4$ lies below $1/x$.

Exercise 5 - Exponential near zero

$$f(0) = 1, f'(0) = 1, \text{ so } L(x) = 1 + x.$$

$$L(0.1) = 1.1 \text{ and } L(-0.05) = 0.95.$$

Final answer: $1+x$; 1.1 and 0.95.

Exercise 6 - Sine near zero

$$L(x) = \sin 0 + \cos 0 \cdot x = x, \text{ so } \sin(0.2) \approx 0.2. \text{ (Exact: } 0.19867.)$$

Final answer: 0.2.

Exercise 7 - Logarithm near one

$$f(1)=0, f'(1)=1, \text{ so } L(x)=x-1.$$

$$L(1.05)=0.05. \text{ (Exact: } 0.04879.)$$

Final answer: 0.05.

Exercise 8 - Tangent near pi over four

$$f\left(\frac{\pi}{4}\right)=1 \text{ and } f'=\sec^2\frac{\pi}{4}=2. \text{ So } L(x)=1+2\left(x-\frac{\pi}{4}\right).$$

$$L(0.8)=1+2(0.8-0.7854)=1.0292. \text{ (Exact: } 1.0296.)$$

Final answer: $\tan(0.8) \approx 1.029$.

Exercise 9 - Overestimate or underestimate, root

$$f''(x)=-\frac{1}{4}x^{-3/2}<0 \text{ for } x>0: \text{ the graph is concave down.}$$

The tangent line lies above the graph, so $L(10)=3+\frac{1}{6}\approx 3.1667$ is greater than $\sqrt{10}\approx 3.1623$.

Final answer: overestimate.

Exercise 10 - Overestimate or underestimate, cube

$$f''(x)=6x>0 \text{ near } 2: \text{ concave up, so the tangent lies below the graph.}$$

$$L(2.1)=8+12(0.1)=9.2 \text{ and the exact value is } 9.261.$$

Final answer: underestimate.

Exercise 11 - Using given values

$$L(x)=7-2(x-3). \text{ Then } L(3.2)=7-0.4=6.6 \text{ and } L(2.9)=7+0.2=7.2.$$

Final answer: 6.6 and 7.2.

Exercise 12 - Linear approximation, a quotient

$$g(1)=\frac{1}{2}. \quad g'(x)=\frac{1}{(x+1)^2}, \text{ so } g'(1)=\frac{1}{4}.$$

$L(x) = \frac{1}{2} + \frac{1}{4}(x-1)$, so $L(1.2) = 0.55$. (Exact: 0.5455.)

Final answer: $g(1.2) \approx 0.55$.

Exercise 13 - Hopital with an exponential and a sine

Substituting gives $\frac{0}{0}$. Differentiate top and bottom: $\frac{3e^{3x}}{2\cos 2x}$.

At $x=0$: $\frac{3}{2}$.

Final answer: $\frac{3}{2}$.

x	-0.01	0.01
quotient	1.4778	1.5228

Numerical check near $x = 0$.

Exercise 14 - Hopital, quadratic

Substituting gives $\frac{0}{0}$. Differentiate: $\frac{2x-4}{2x}$, which at $x=1$ is $\frac{-2}{2} = -1$.

Final answer: -1 .

Exercise 15 - Hopital at x equals one

Substituting gives $\frac{0}{0}$. Differentiate: $\frac{1/x}{2x} = \frac{1}{2x^2}$.

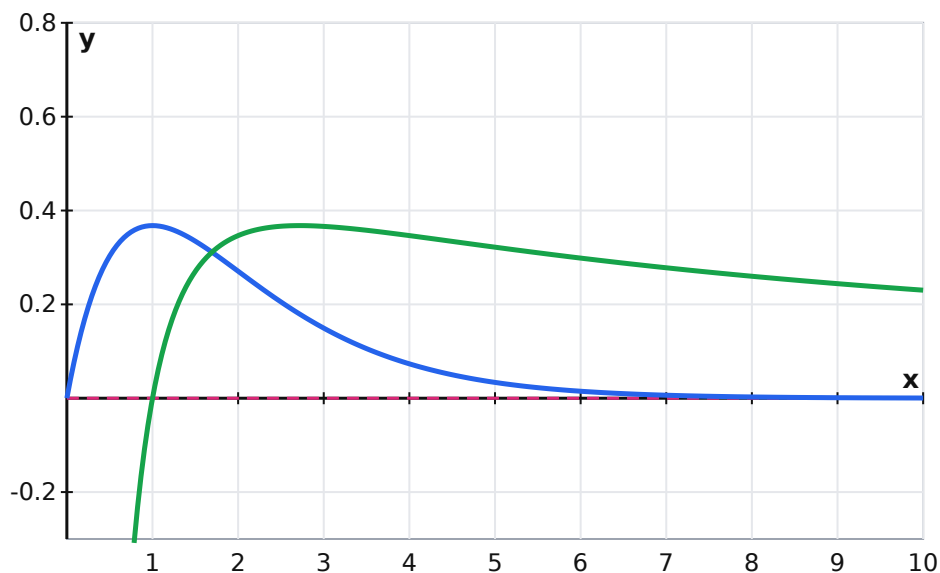
At $x=1$ this is $\frac{1}{2}$.

Final answer: $\frac{1}{2}$.

Exercise 16 - Hopital, infinity over infinity

- a. $\frac{\infty}{\infty}$: $\lim_{x \rightarrow \infty} \frac{1}{e^x} = 0$.
- b. $\frac{\infty}{\infty}$: $\lim_{x \rightarrow \infty} \frac{1/x}{1} = 0$.

Final answer: 0 and 0.



Both quotients approach 0 as x grows.

Exercise 17 - Hopital twice

$\frac{\infty}{\infty}$. First: $\frac{6x}{2x+5}$ (still $\frac{\infty}{\infty}$). Second: $\frac{6}{2} = 3$.

Final answer: 3.

Exercise 18 - Three applications

$\frac{0}{0}$. Differentiate: $\frac{\cos x - 1}{3x^2}$ (still $\frac{0}{0}$), then $\frac{-\sin x}{6x}$ (still $\frac{0}{0}$), then $\frac{-\cos x}{6} \rightarrow -\frac{1}{6}$.

Final answer: $-\frac{1}{6}$.

Exercise 19 - When not to use the rule

Substituting gives $\frac{2}{-1} = -2$, which is not $\frac{0}{0}$ or $\frac{\infty}{\infty}$. Applying the rule would give $\frac{1}{1} = 1$, which is wrong.

Final answer: -2 .

Exercise 20 - Square garden

From the earlier work, $\sqrt{26} \approx 5.1$. The square root function is concave down, so the tangent line is above the curve.

Final answer: about 5.1 ft (a slight overestimate; the exact side is about 5.099 ft).

Exercise 21 - Change in volume

$$V'(r) = 4\pi r^2, \text{ so } V'(5) = 100\pi.$$

$$\Delta V \approx V'(5) \cdot 0.1 = 10\pi \approx 31.4.$$

The exact change is about 32.

Final answer: $\Delta V \approx 10\pi \approx 31.4 \text{ cm}^3$.

Exercise 22 - Measurement error

$$A'(s) = 2s, \text{ so } A'(10) = 20. \text{ Then } \Delta A \approx 20(0.1) = 2.$$

Final answer: about 2 cm^2 .

Exercise 23 - Pizza area

$$A'(8) = 2\pi(8) = 16\pi, \text{ so } \Delta A \approx 16\pi(0.2) = 3.2\pi \approx 10.05.$$

Final answer: about 10.05 in^2 .

Exercise 24 - Temperature estimate

$$T(12) \approx 350 + 4(2) = 358 \text{ }^\circ\text{F}.$$

The sign of T'' would decide it: concave down ($T'' < 0$) gives an overestimate, concave up an underestimate.

Final answer: 358°F .