

AP Calculus AB - Linearization and L'Hopital's Rule - Worksheet

Common Core AP CALC AB 4.5, AP CALC AB 4.6, AP CALC AB 4.7 · maths-pdf.fr/en/

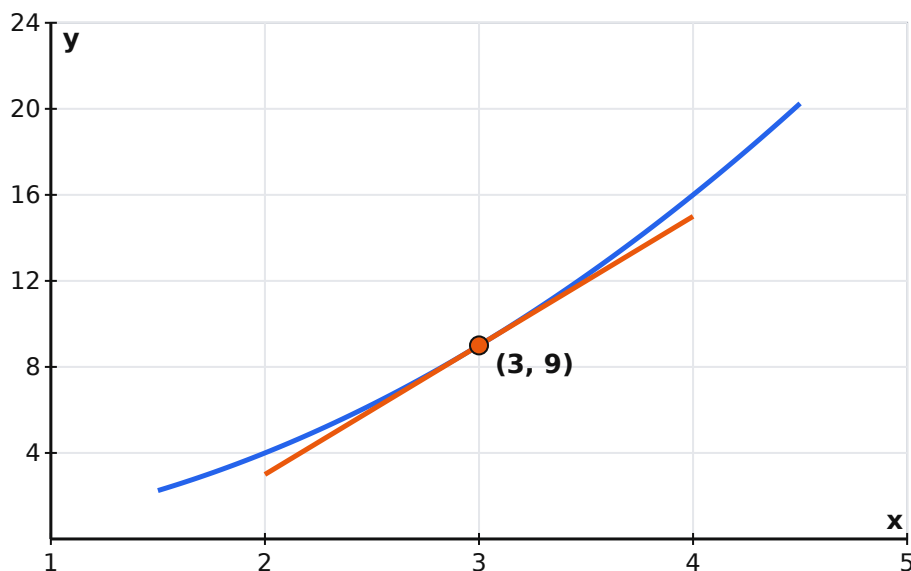
Name: _____ Class: _____ Date: ____/____/____

These 24 problems practice local linear approximation, over- and underestimates from concavity, and L'Hopital's Rule for indeterminate limits (AP Calculus AB, topics 4.5, 4.6, 4.7). The early problems build the basic skill, and each one uses a different function or situation so that no two feel alike.

Later problems use over- and underestimates, tables of values, repeated applications of the rule and real situations such as measurement error and a pizza area. Work in pencil, show every step, and check the answer key only after a full attempt.

AP Calculus AB · Common Core State Standards: **AP CALC AB 4.5, AP CALC AB 4.6, AP CALC AB 4.7**

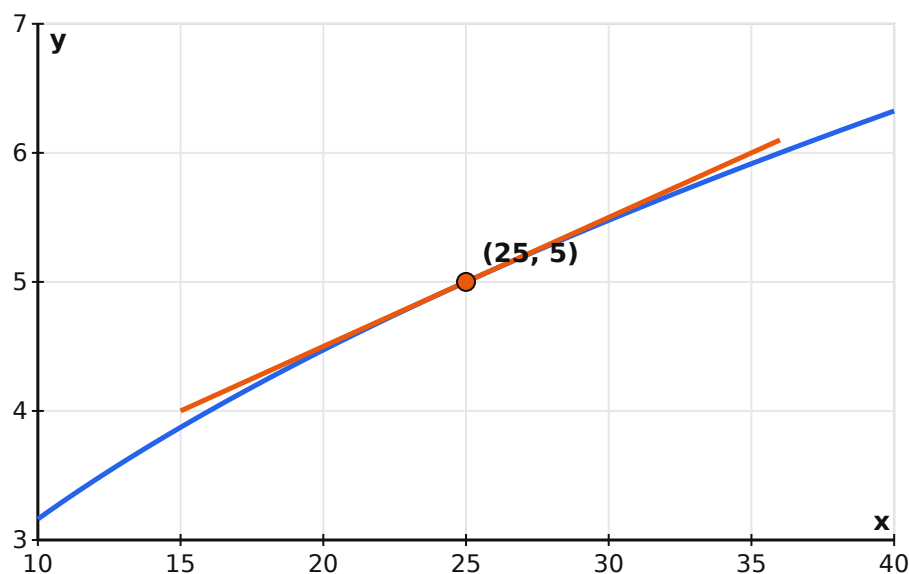
Exercise 1 - Linearization of x squared



The tangent to $y = x^2$ at $x = 3$.

Find the linearization of $f(x) = x^2$ at $a = 3$ and use it to estimate $(3.1)^2$. Compare with the exact value.

Exercise 2 - Estimating a square root



Tangent to $\sqrt{x}$ at $x = 25$.

Use a linear approximation of $f(x) = \sqrt{x}$ at $a = 25$ to estimate $\sqrt{26}$.

Exercise 3 - Estimating a cube root

Estimate $\sqrt[3]{28}$ using the linearization of $f(x) = \sqrt[3]{x}$ at $a = 27$.

Exercise 4 - Estimating a reciprocal

Estimate $\frac{1}{2.1}$ using the linearization of $f(x) = \frac{1}{x}$ at $a = 2$.

Exercise 5 - Exponential near zero

Find the linearization of $f(x) = e^x$ at $a = 0$ and estimate $e^{0.1}$ and $e^{-0.05}$.

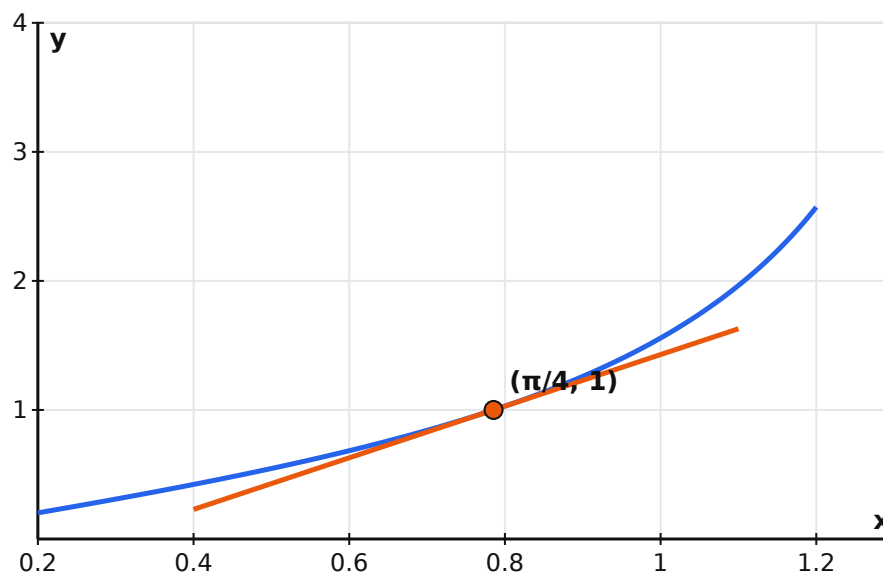
Exercise 6 - Sine near zero

Estimate $\sin(0.2)$ using the linearization of $\sin x$ at 0.

Exercise 7 - Logarithm near one

Estimate $\ln(1.05)$ with the linearization of $\ln x$ at $a = 1$.

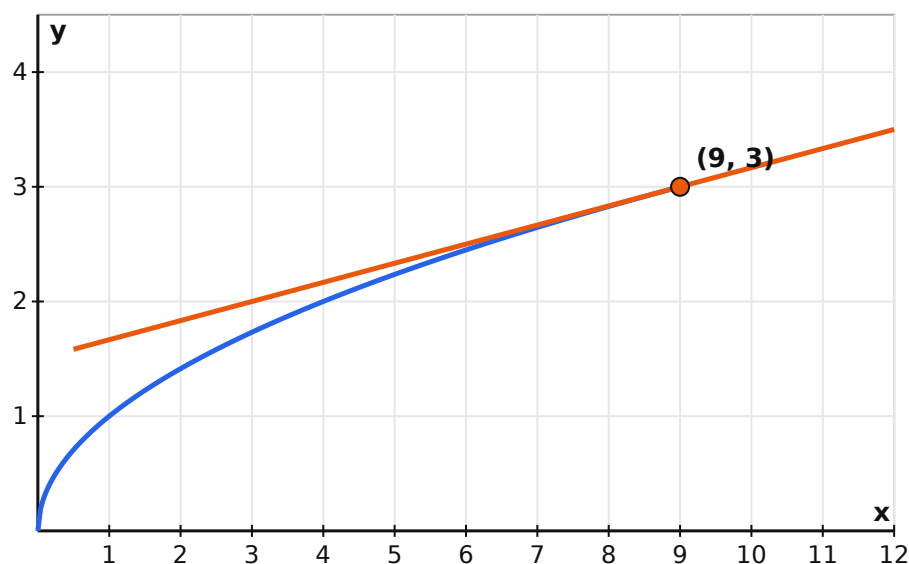
Exercise 8 - Tangent near pi over four



The tangent to $\tan x$ at $x = \pi/4$.

Estimate $\tan(0.8)$ using the linearization of $f(x) = \tan x$ at $a = \frac{\pi}{4}$.

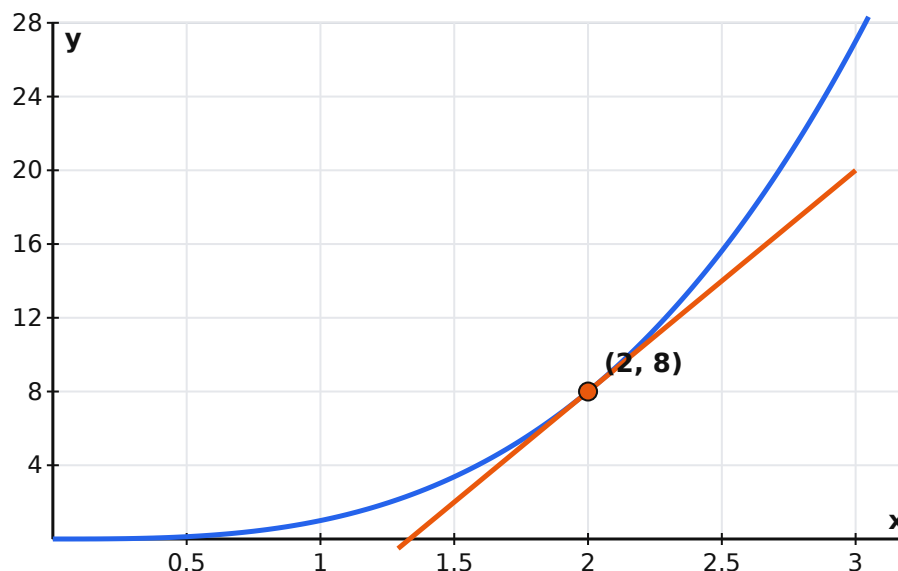
Exercise 9 - Overestimate or underestimate, root



The tangent line lies above $y = \sqrt{x}$.

The linearization of $f(x) = \sqrt{x}$ at $a = 9$ is used to estimate $\sqrt{10}$. Is the estimate an overestimate or an underestimate? Justify.

Exercise 10 - Overestimate or underestimate, cube



The tangent line lies below $y = x^3$.

The linearization of $f(x) = x^3$ at $a = 2$ is used to estimate $(2.1)^3$. Over or under?

Exercise 11 - Using given values

A differentiable function has $f(3) = 7$ and $f'(3) = -2$. Estimate $f(3.2)$ and $f(2.9)$.

Exercise 12 - Linear approximation, a quotient

Find the linearization of $g(x) = \frac{x}{x+1}$ at $a = 1$ and estimate $g(1.2)$.

Exercise 13 - Hopital with an exponential and a sine

Evaluate $\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{\sin 2x}$.

Exercise 14 - Hopital, quadratic

Evaluate $\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x^2 - 1}$.

Exercise 15 - Hopital at x equals one

Evaluate $\lim_{x \rightarrow 1} \frac{\ln x}{x^2 - 1}$.

Exercise 16 - Hopital, infinity over infinity

Evaluate $\lim_{x \rightarrow \infty} \frac{x}{e^x}$ and $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$.

Exercise 17 - Hopital twice

Evaluate $\lim_{x \rightarrow \infty} \frac{3x^2+1}{x^2+5x}$ with L'Hôpital's Rule.

Exercise 18 - Three applications

Evaluate $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$.

Exercise 19 - When not to use the rule

Explain why L'Hôpital's Rule should not be used for $\lim_{x \rightarrow 0} \frac{x+2}{x-1}$ and find the limit.

Exercise 20 - Square garden

Mei wants a square garden with an area of 26 square feet. Use a linear approximation of $\sqrt{x}$ at 25 to estimate the side length, and decide if your estimate is high or low.

Exercise 21 - Change in volume

A spherical balloon has radius 5 cm, which grows to 5.1 cm. Use the linearization of $V(r) = \frac{4}{3}\pi r^3$ at $r=5$ to estimate the change in volume.

Exercise 22 - Measurement error

A square tile's side is measured as 10 cm with a possible error of 0.1 cm. Use a linear approximation to estimate the maximum error in the area $A = s^2$.

Exercise 23 - Pizza area

A pizza's radius grows from 8 in to 8.2 in. Use $A = \pi r^2$ and a linear approximation to estimate the change in area.

Exercise 24 - Temperature estimate

The temperature of an oven in °F is $T(t)$ with $T(10) = 350$ and $T'(10) = 4$ °F per minute. Estimate $T(12)$, and say what extra information would tell you whether it is an over- or underestimate.