

AP Calculus AB - Linearization and L'Hopital's Rule - Test Answer Key

Common Core AP CALC AB 4.5, AP CALC AB 4.6, AP CALC AB 4.7 · maths-pdf.fr/en/

Use this answer key to score the linearization test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

AP Calculus AB · Common Core State Standards: **AP CALC AB 4.5, AP CALC AB 4.6, AP CALC AB 4.7**

Question 1 - Linearization [2 pts]

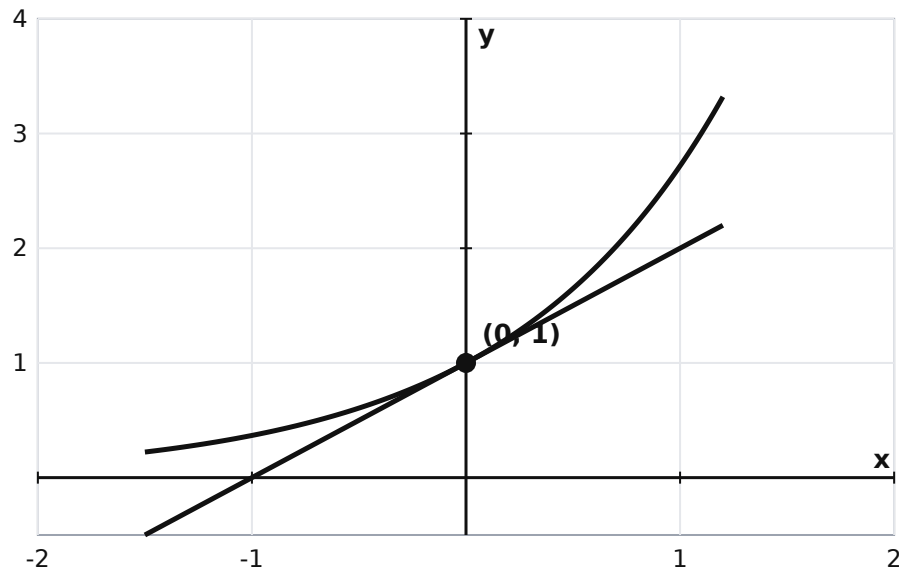
$$f(16) = 4, f'(16) = \frac{1}{8}. L(x) = 4 + \frac{1}{8}(x - 16).$$

$$L(17) = 4.125. \text{ Final answer: } \sqrt{17} \approx 4.125.$$

Scoring: 1 point for $L(x)$; 1 point for the estimate.

Question 2 - Tangent line estimate [3 pts]

- a. $L(x) = 1 + x$.
- b. $L(0.2) = 1.2$ (exact 1.2214).
- c. $f''(x) = e^x > 0$: concave up, so the tangent lies below the curve. **Underestimate.**

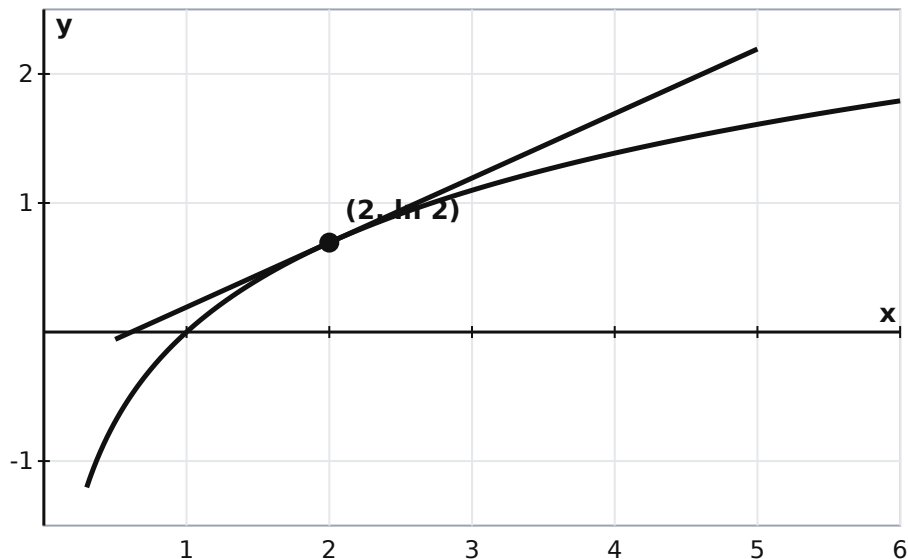


The graph of e^x and its tangent line at $x = 0$.

Scoring: 1 point per part.

Question 3 - Over or under [2 pts]

$f''(x) = -\frac{1}{x^2} < 0$: concave down, so the tangent line lies above the graph and the estimate is **too high**.



The graph of $\ln x$ and its tangent at $x = 2$.

Scoring: 1 point for the second derivative; 1 point for the conclusion.

Question 4 - Using a table [3 pts]

- a. $L(4.5) = 10 - 3(0.5) = 8.5$.
- b. The table gives 8.6, so the estimate is 0.1 too low.
- c. The tangent line is below the graph, so **concave up**.

Scoring: 1 point per part.

Question 5 - Hopital 0 over 0, second form [2 pts]

$\frac{0}{0}$. Differentiate: $\frac{3x^2}{2x} = \frac{27}{6} = \frac{9}{2}$.

Final answer: $\frac{9}{2}$.

Scoring: 1 point for identifying 0/0 and differentiating; 1 point for the value.

Question 6 - Hopital infinity form [3 pts]

$\frac{\infty}{\infty}$. First: $\frac{2\ln x \cdot \frac{1}{x}}{1} = \frac{2\ln x}{x}$ (still $\frac{\infty}{\infty}$). Second: $\frac{2/x}{1} = \frac{2}{x} \rightarrow 0$.

Final answer: 0.

Scoring: 1 point for each application; 1 point for the value.

Question 7 - Error estimation [2 pts]

$A = s^2$, $A' = 2s = 48$. $\Delta A \approx 48(0.25) = 12$.

Final answer: **about 12 square inches**.

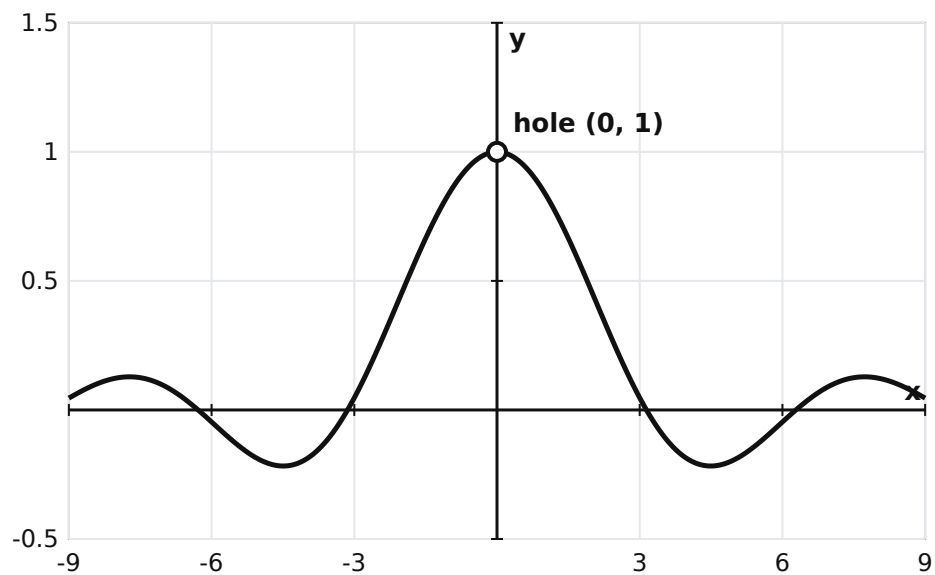
Scoring: 1 point for A' ; 1 point for the estimate with units.

Question 8 - Limit by two methods [3 pts]

a. $\frac{0}{0}$: $\frac{\cos x}{1} \rightarrow 1$.

b. $\frac{0}{0}$: $\frac{\sec^2 x}{1} \rightarrow 1$.

c. Substituting gives $\frac{1}{1} = 1$, which is not indeterminate; the limit is simply 1.



The graph of $\sin x / x$.

Scoring: 1 point per part.

Total
20 points