

AP Calculus AB - Mean Value Theorem and Extreme Value Theorem - Lesson

Common Core AP CALC AB 5.1, AP CALC AB 5.2, AP CALC AB 5.3 · maths-pdf.fr/en/

The Mean Value Theorem says that a function continuous on $[a, b]$ and differentiable on (a, b) has a point c where the instantaneous rate $f'(c)$ equals the average rate $(f(b) - f(a))/(b - a)$. The Extreme Value Theorem guarantees an absolute maximum and minimum on a closed interval. Absolute extrema occur at endpoints or critical numbers.

AP Calculus AB · Common Core State Standards: **AP CALC AB 5.1, AP CALC AB 5.2, AP CALC AB 5.3**

Key facts

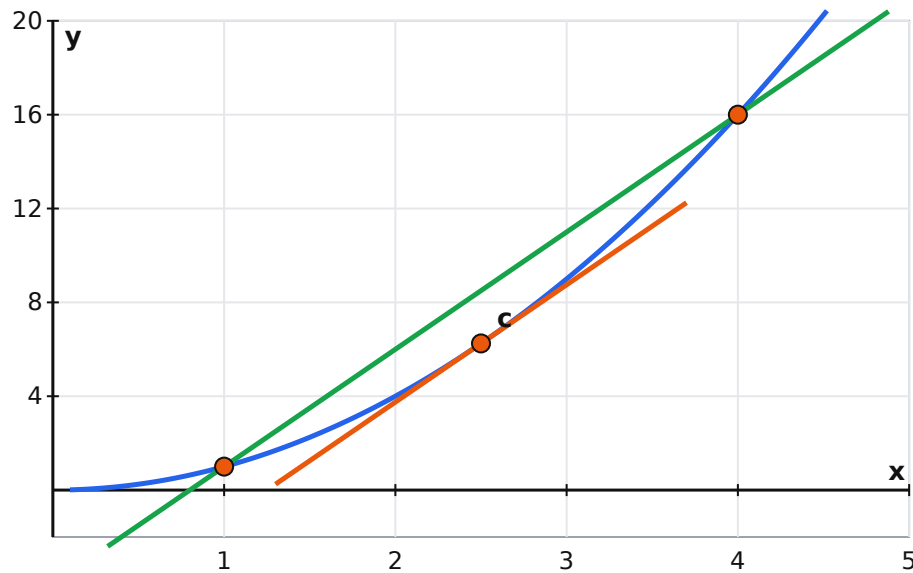
- MVT: $f'(c) = (f(b) - f(a))/(b - a)$ for some c in (a, b) , if f is continuous and differentiable.
- EVT: a continuous function on a closed interval has an absolute maximum and an absolute minimum.
- Critical numbers are where $f' = 0$ or f' is undefined (inside the domain).
- Candidates test: compare f at endpoints and critical numbers.

What is the Mean Value Theorem?

If you drive 130 miles in 2 hours, your average speed is 65 mph. The Mean Value Theorem says that at some instant your speedometer read exactly 65 mph, provided the motion was continuous and smooth.

Mean Value Theorem (MVT)

If f is continuous on $[a, b]$ and differentiable on (a, b) , then there is at least one c in (a, b) with $f'(c) = \frac{f(b) - f(a)}{b - a}$.



The secant slope is 5; the tangent at $c = 2.5$ is parallel to it.

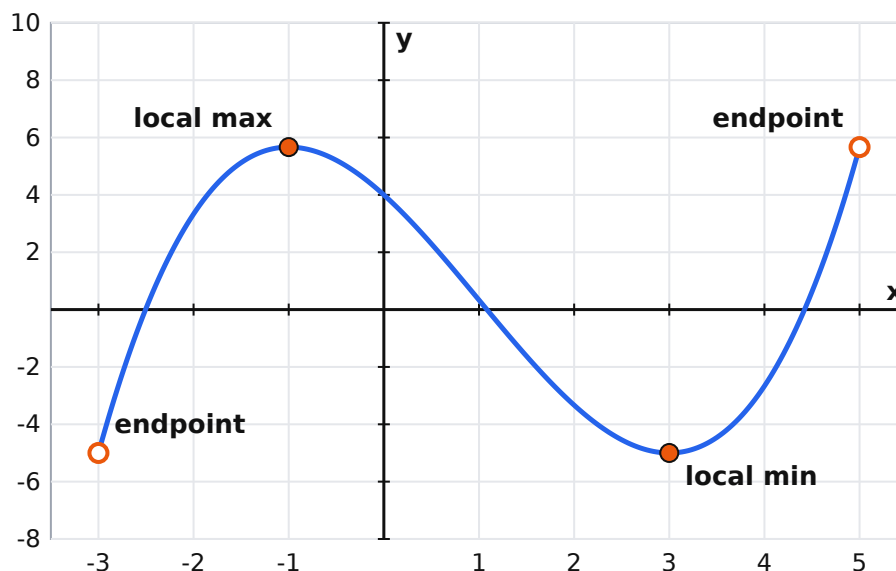
Example. For $f(x) = x^2$ on $[1, 4]$, the average rate is $\frac{16-1}{3} = 5$. Solve $2c = 5$: $c = 2.5$, which is inside $(1, 4)$. To use the theorem on an exam, state that f is continuous and differentiable, then compute the average rate.

What is the Extreme Value Theorem?

Extreme Value Theorem (EVT)

If f is continuous on a closed interval $[a, b]$, then f has an absolute maximum and an absolute minimum on $[a, b]$.

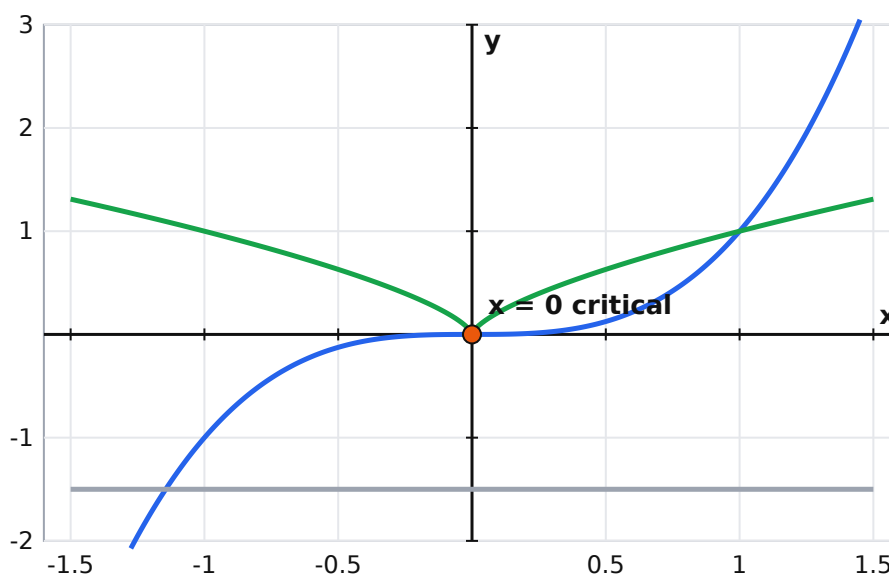
Both hypotheses matter: the interval must be closed and the function must be continuous. For example, $f(x) = x$ on $(0, 1)$ has no maximum or minimum because the endpoints are excluded.



A continuous function on $[a, b]$ attains its extreme values at endpoints or critical points.

What are critical numbers?

A **critical number** of f is an input c in the domain where $f'(c) = 0$ or $f'(c)$ does not exist. Extreme values inside an interval happen only at critical numbers.



Critical numbers: $f'(c) = 0$ (blue) or $f'(c)$ undefined (green cusp).

How do you find absolute extrema on a closed interval?

1. Check that f is continuous on $[a, b]$.
2. Find all critical numbers inside (a, b) .
3. Evaluate f at the critical numbers and at both endpoints.
4. The largest value is the absolute maximum and the smallest is the absolute minimum.

x	-2	-1	1	3
f(x)	-2	2	-2	18
type	endpoint	critical	critical	endpoint

List endpoints and critical numbers, then compare values.

Here the largest value is 18 at $x=3$ and the smallest is -2 , reached at both $x=-2$ and $x=1$.

Questions about mean value theorem and extreme value theorem

What does the Mean Value Theorem say?

If f is continuous on $[a, b]$ and differentiable on (a, b) , some point c in (a, b) has a tangent slope equal to the average slope of the secant line from a to b .

What are the conditions for the Mean Value Theorem?

The function must be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) . If either fails, the conclusion may fail.

What is a critical number?

A critical number is an input c in the domain of f where $f'(c) = 0$ or $f'(c)$ does not exist. Interior extreme values occur only at critical numbers.

How do you find absolute extrema on a closed interval?

Find the critical numbers inside the interval, evaluate f at those numbers and at the two endpoints, and then compare the values. The largest is the maximum and the smallest is the minimum.

Does every function have an absolute maximum?

No. The Extreme Value Theorem needs a continuous function on a closed interval. On an open interval or with a discontinuity, there might be no maximum or minimum.