

AP Calculus AB - Mean Value Theorem and Extreme Value Theorem - Answer Key

Common Core AP CALC AB 5.1, AP CALC AB 5.2, AP CALC AB 5.3 · maths-pdf.fr/en/

This answer key gives a full worked solution for every problem in the mean value theorem worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 5.1, AP CALC AB 5.2, AP CALC AB 5.3**

Exercise 1 - MVT with a parabola

f is a polynomial, so it is continuous and differentiable everywhere.

Average rate: $\frac{16-1}{4-1}=5$. Solve $f'(c)=2c=5$: $c=2.5$, which is in $(1,4)$.

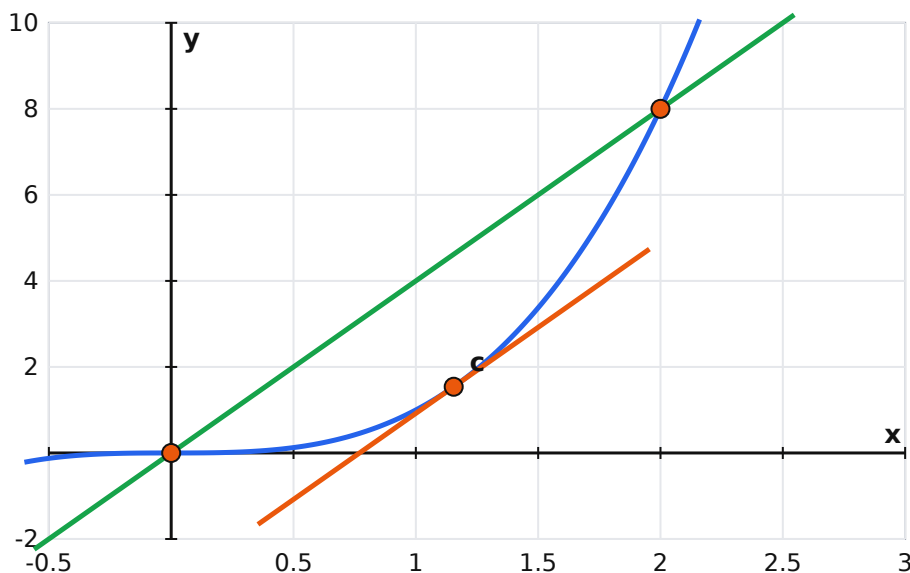
Final answer: $c=\frac{5}{2}$.

Exercise 2 - MVT with a cubic

Average rate: $\frac{8-0}{2}=4$. Solve $3c^2=4$: $c=\pm\frac{2}{\sqrt{3}}$.

Only $\frac{2}{\sqrt{3}}\approx 1.155$ lies in $(0,2)$.

Final answer: $c=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$.



The tangent at $c \approx 1.155$ is parallel to the secant.

Exercise 3 - MVT with a logarithm

Average rate: $\frac{\ln e^2 - \ln 1}{e^2 - 1} = \frac{2}{e^2 - 1}$.

Solve $\frac{1}{c} = \frac{2}{e^2 - 1}$: $c = \frac{e^2 - 1}{2} \approx 3.19$, which lies in $(1, e^2)$.

Final answer: $c = \frac{e^2 - 1}{2}$.

Exercise 4 - MVT with a reciprocal

Average rate: $\frac{\frac{1}{4} - 1}{3} = -\frac{1}{4}$. Solve $-\frac{1}{c^2} = -\frac{1}{4}$: $c^2 = 4$, so $c = 2$ (since $c > 0$).

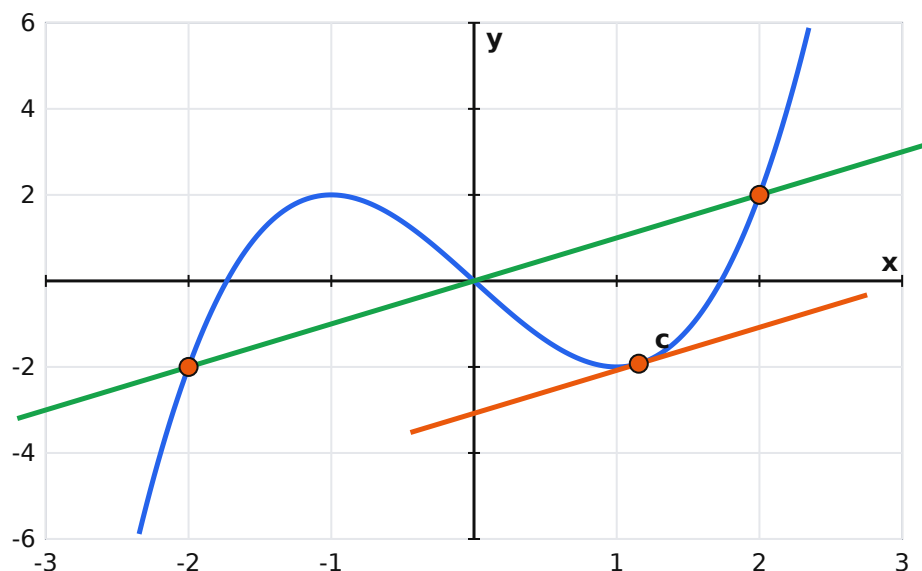
Final answer: $c = 2$.

Exercise 5 - Two values of c

$f(2) = 2$ and $f(-2) = -2$, so the average rate is $\frac{2 - (-2)}{4} = 1$.

Solve $3c^2 - 3 = 1$: $c^2 = \frac{4}{3}$, so $c = \pm \frac{2}{\sqrt{3}}$, both in $(-2, 2)$.

Final answer: $c = \pm \frac{2\sqrt{3}}{3}$.



The tangent at $c \approx 1.155$ (another is at -1.155).

Exercise 6 - When the MVT fails

The average rate is $\frac{2-1}{3} = \frac{1}{3}$. But f is not differentiable at $x=0$, which is inside the interval, so the MVT does not apply.

Indeed $f'(x) = -1$ for $x < 0$ and 1 for $x > 0$, never $\frac{1}{3}$.

Final answer: No. The theorem does not apply, and no such c exists.

Exercise 7 - Discontinuity

f is undefined at $x=0$, so it is not continuous on $[-1, 1]$. Also $f'(x) = -\frac{1}{x^2} < 0$ everywhere it exists, while the average rate is $\frac{1-(-1)}{2} = 1 > 0$.

Final answer: not continuous on $[-1, 1]$.

Exercise 8 - Speeding ticket

Distance: $130 - 25 = 105$ miles in 1.5 hours. Average speed: $\frac{105}{1.5} = 70$ mph.

Position is continuous and differentiable, so by the MVT the speed was exactly 70 mph at some instant, which exceeds 65 .

Final answer: 70 mph average, so speeding at some instant.

Exercise 9 - MVT from a table

Average rate: $\frac{11-3}{4}=2$. The MVT guarantees a c in $(0,4)$ with $f'(c)=2$.

Therefore $f'(x)$ cannot be less than 2 everywhere.

Final answer: $f'(c)=2$ for some c ; no.

Exercise 10 - Rolle's theorem

$f(1)=0$ and $f(3)=0$, so the average rate is 0. $f'(c)=2c-4=0$ gives $c=2$.

Final answer: $c=2$.

Exercise 11 - EVT with a parabola

$f'=2x=0$ gives the critical number $x=0$. Candidates: $f(-1)=1$, $f(0)=0$, $f(3)=9$.

Final answer: absolute maximum 9 at $x=3$; absolute minimum 0 at $x=0$.

Exercise 12 - Critical numbers with a Gaussian factor

Product and chain rules: $f'(x)=e^{-x^2/2}+x\cdot e^{-x^2/2}(-x)=e^{-x^2/2}(1-x^2)$.

The exponential is never zero, so solve $1-x^2=0$. The derivative exists everywhere.

Final answer: $x=-1$ and $x=1$.

Exercise 13 - Undefined derivative

$f'(x)=\frac{2}{3}x^{-1/3}=\frac{2}{3\sqrt[3]{x}}$ is never zero, but is undefined at $x=0$, which is in the domain.

Final answer: $x=0$.

Exercise 14 - Product with an exponential

$f'(x)=e^{-x}-xe^{-x}=e^{-x}(1-x)$. Since $e^{-x}>0$, set $1-x=0$.

Final answer: $x=1$.

Exercise 15 - Trig critical numbers

$$f' = \cos x - \sin x = 0 \text{ means } \tan x = 1.$$

On $(0, 2\pi)$: $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$.

Final answer: $x = \frac{\pi}{4}, \frac{5\pi}{4}$.

Exercise 16 - Closed interval method, cubic

$$f' = 3x^2 - 3 = 0 \text{ gives } x = \pm 1, \text{ both in the interval.}$$

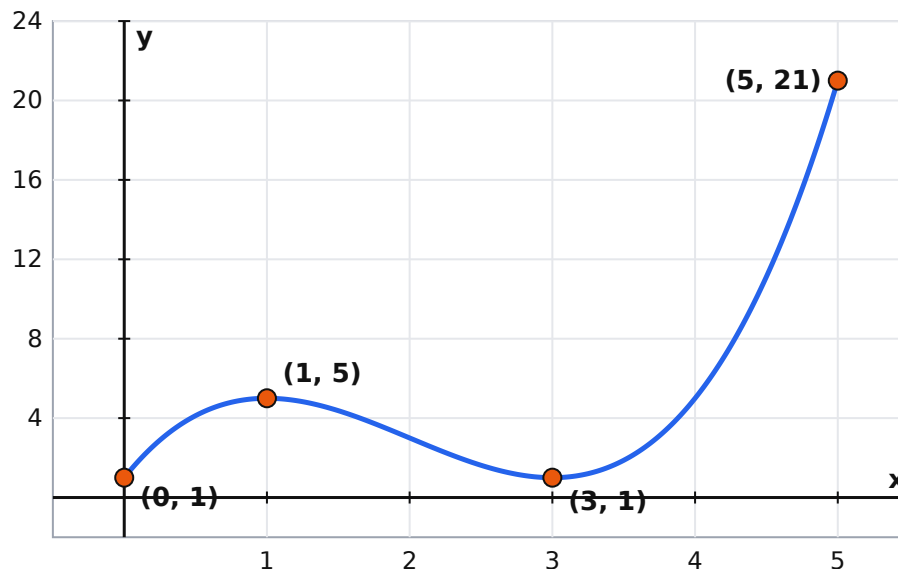
$$f(-2) = -2, f(-1) = 2, f(1) = -2, f(3) = 18.$$

Final answer: max 18 at $x=3$; min -2 at $x=-2$ and $x=1$.

Exercise 17 - Closed interval method, second cubic

$$f' = 3x^2 - 12x + 9 = 3(x-1)(x-3). \text{ Candidates: } f(0)=1, f(1)=5, f(3)=1, f(5)=21.$$

Final answer: max 21 at $x=5$; min 1 at $x=0$ and $x=3$.



The candidates on $[0, 5]$.

Exercise 18 - Closed interval method, rational

$f' = \frac{1-x^2}{(x^2+1)^2}$, zero at $x=1$. Candidates: $f(0)=0$, $f(1)=\frac{1}{2}$, $f(4)=\frac{4}{17}$.

Final answer: $\max \frac{1}{2}$ at $x=1$; $\min 0$ at $x=0$.

Exercise 19 - Endpoint critical number

$f' = \frac{1}{x} - 1 = 0$ gives $x=1$, an endpoint. Candidates: $f(1)=-1$ and $f(4)=\ln 4 - 4 \approx -2.614$.

Final answer: $\max -1$ at $x=1$; $\min \ln 4 - 4$ at $x=4$.

Exercise 20 - Greenhouse temperature

$T' = -2t + 12 = 0$ gives $t=6$. Candidates: $T(0)=40$, $T(6)=76$, $T(10)=60$.

Final answer: highest 76°F at $t=6$; lowest 40°F at $t=0$.

Exercise 21 - Maximum revenue

$R' = 120 - 2x = 0$ gives $x=60$. Candidates: $R(0)=0$, $R(60)=3600$, $R(100)=2000$.

Final answer: 60 boards, revenue \$3600.

Exercise 22 - Parallel tangent

Average rate: $\frac{6-0}{2}=3$. $3c^2-1=3$ gives $c=\frac{2}{\sqrt{3}}$.

$f(c) = \frac{8}{3\sqrt{3}} - \frac{2}{\sqrt{3}} = \frac{2}{3\sqrt{3}}$. Tangent: $y - \frac{2}{3\sqrt{3}} = 3\left(x - \frac{2}{\sqrt{3}}\right)$.

Final answer: $c = \frac{2}{\sqrt{3}} \approx 1.155$.