

AP Calculus AB - Mean Value Theorem and Extreme Value Theorem - Worksheet

Common Core AP CALC AB 5.1, AP CALC AB 5.2, AP CALC AB 5.3 · maths-pdf.fr/en/

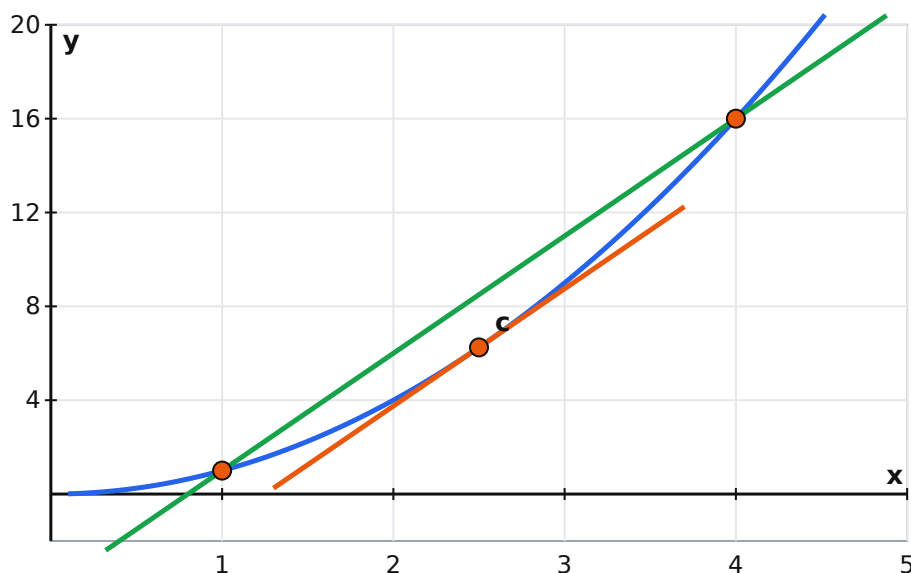
Name: _____ Class: _____ Date: ____/____/____

These 22 problems practice the Mean Value Theorem, the Extreme Value Theorem, critical numbers and absolute extrema on closed intervals (AP Calculus AB, topics 5.1, 5.2, 5.3). The early problems build the basic skill, and each one uses a different function or situation so that no two feel alike.

Later problems cover hypotheses that fail, the closed interval method, and situations such as a speeding ticket, greenhouse temperature and shop revenue. Work in pencil, show every step, and check the answer key only after a full attempt.

AP Calculus AB · Common Core State Standards: **AP CALC AB 5.1, AP CALC AB 5.2, AP CALC AB 5.3**

Exercise 1 - MVT with a parabola



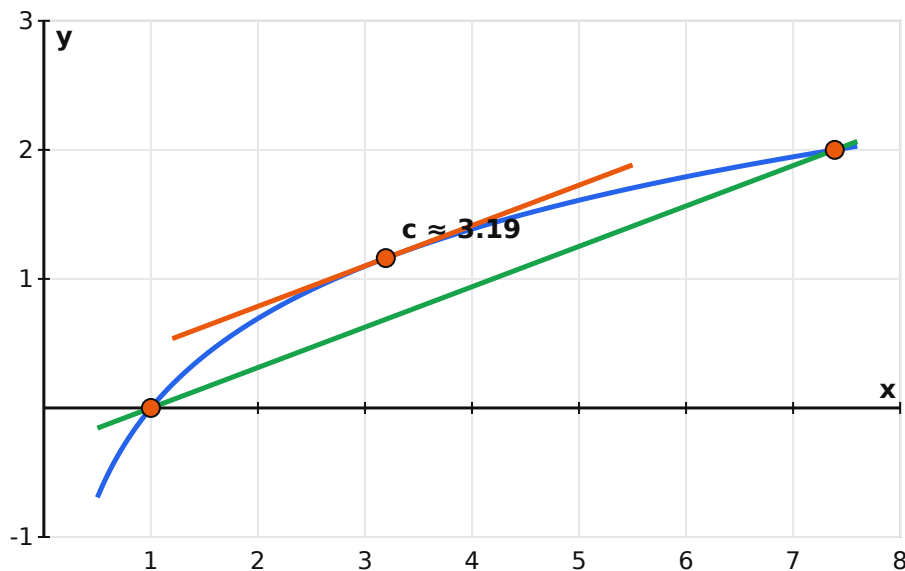
The secant from $x = 1$ to $x = 4$.

Verify the Mean Value Theorem for $f(x) = x^2$ on $[1, 4]$ and find c .

Exercise 2 - MVT with a cubic

Find all c guaranteed by the MVT for $f(x) = x^3$ on $[0, 2]$.

Exercise 3 - MVT with a logarithm



The tangent at c is parallel to the secant.

Find the value c guaranteed by the MVT for $f(x) = \ln x$ on $[1, e^2]$.

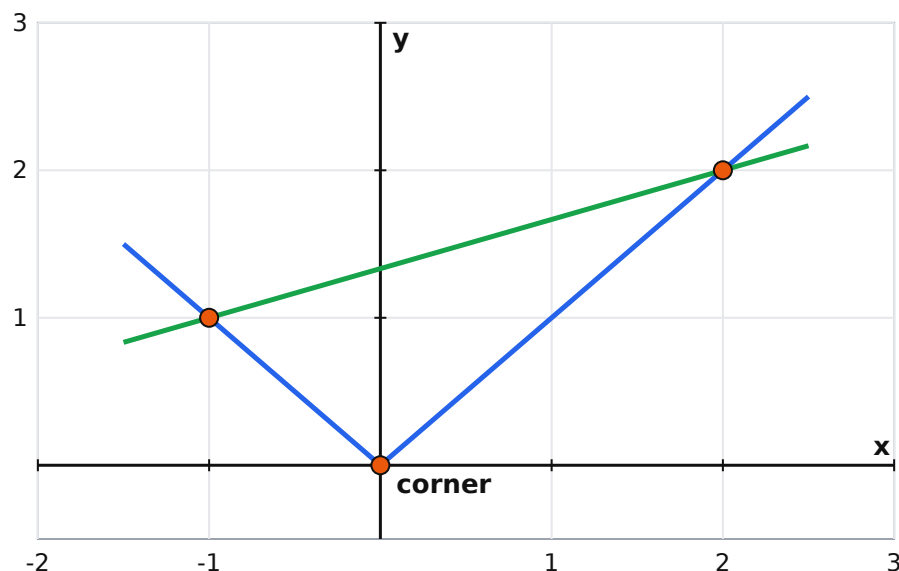
Exercise 4 - MVT with a reciprocal

Find c for $f(x) = \frac{1}{x}$ on $[1, 4]$.

Exercise 5 - Two values of c

Find all c guaranteed by the MVT for $f(x) = x^3 - 3x$ on $[-2, 2]$.

Exercise 6 - When the MVT fails



The graph of $|x|$ on $[-1, 2]$.

Does the MVT apply to $f(x) = |x|$ on $[-1, 2]$? Is there a c with $f'(c) = \frac{1}{3}$?

Exercise 7 - Discontinuity

Explain why the MVT does not apply to $f(x) = \frac{1}{x}$ on $[-1, 1]$.

Exercise 8 - Speeding ticket

A car passes a mile marker 25 at 2:00 p.m. and mile marker 130 at 3:30 p.m. The speed limit is 65 mph. Use the MVT to show that the driver was speeding at some moment.

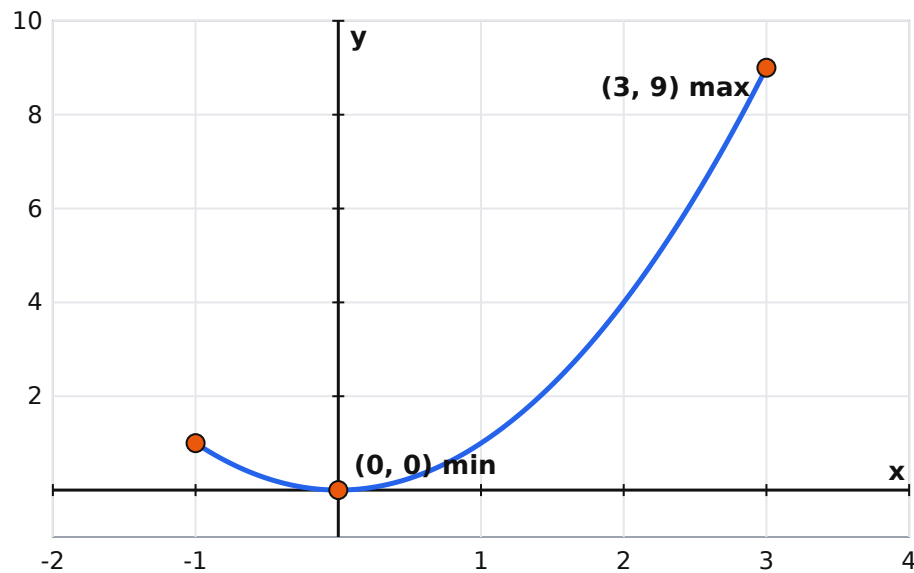
Exercise 9 - MVT from a table

A differentiable function has $f(0) = 3$ and $f(4) = 11$. What does the MVT guarantee? Could $f'(x)$ be less than 2 for all x in $(0, 4)$?

Exercise 10 - Rolle's theorem

Find the c guaranteed for $f(x) = x^2 - 4x + 3$ on $[1, 3]$, where $f(1) = f(3)$.

Exercise 11 - EVT with a parabola



The graph of x^2 on $[-1, 3]$.

Find the absolute maximum and minimum of $f(x) = x^2$ on $[-1, 3]$.

Exercise 12 - Critical numbers with a Gaussian factor

Find the critical numbers of $f(x) = xe^{-x^2/2}$.

Exercise 13 - Undefined derivative

Find the critical numbers of $f(x) = x^{2/3}$.

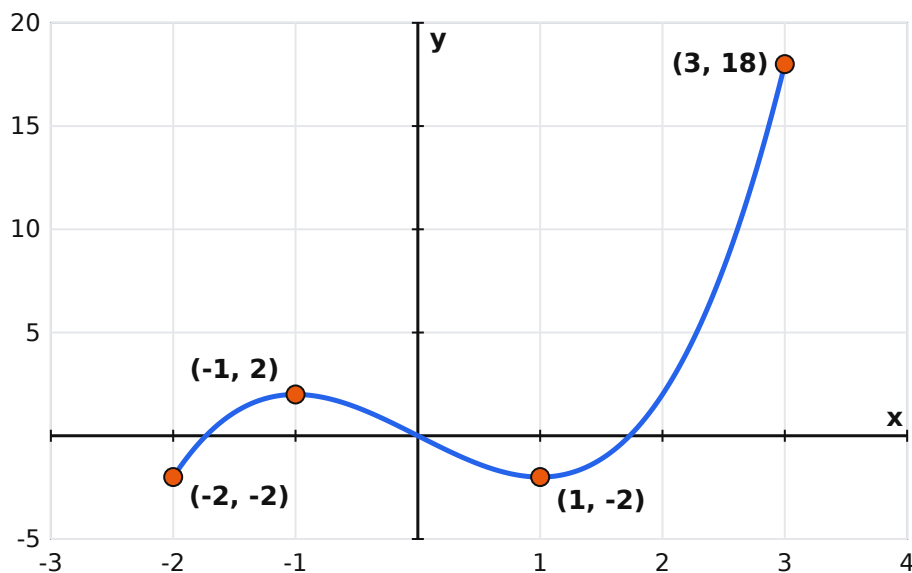
Exercise 14 - Product with an exponential

Find the critical numbers of $f(x) = xe^{-x}$.

Exercise 15 - Trig critical numbers

Find the critical numbers of $f(x) = \sin x + \cos x$ on $(0, 2\pi)$.

Exercise 16 - Closed interval method, cubic



Candidates for extrema on $[-2, 3]$.

Find the absolute extrema of $f(x) = x^3 - 3x$ on $[-2, 3]$.

Exercise 17 - Closed interval method, second cubic

Find the absolute extrema of $f(x) = x^3 - 6x^2 + 9x + 1$ on $[0, 5]$.

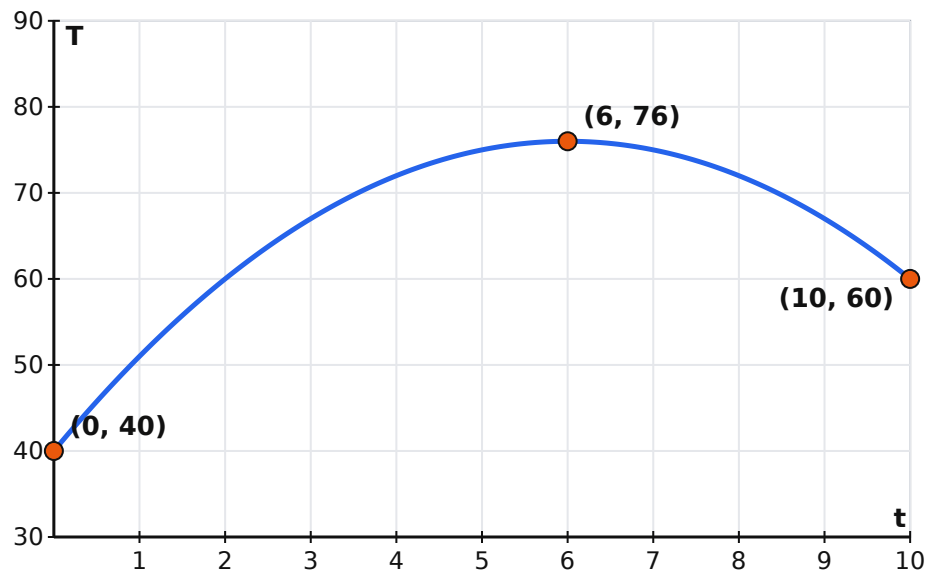
Exercise 18 - Closed interval method, rational

Find the absolute extrema of $f(x) = \frac{x}{x^2 + 1}$ on $[0, 4]$.

Exercise 19 - Endpoint critical number

Find the absolute extrema of $f(x) = \ln x - x$ on $[1, 4]$.

Exercise 20 - Greenhouse temperature



Greenhouse temperature in $^{\circ}\text{F}$.

The temperature in a greenhouse in $^{\circ}\text{F}$ is $T(t) = -t^2 + 12t + 40$ for $0 \leq t \leq 10$ hours. Find the highest and lowest temperatures.

Exercise 21 - Maximum revenue

A skate shop's revenue in dollars from selling x boards a week is $R(x) = 120x - x^2$ for $0 \leq x \leq 100$. Find the number of boards that gives the greatest revenue and the revenue.

Exercise 22 - Parallel tangent

For $f(x) = x^3 - x$ on $[0, 2]$, find the value c where the tangent is parallel to the secant line, and write the equation of that tangent line.