

AP Calculus AB - Mean Value Theorem and Extreme Value Theorem - Test Answer Key

Common Core AP CALC AB 5.1, AP CALC AB 5.2, AP CALC AB 5.3 · maths-pdf.fr/en/

Use this answer key to score the mean value theorem test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

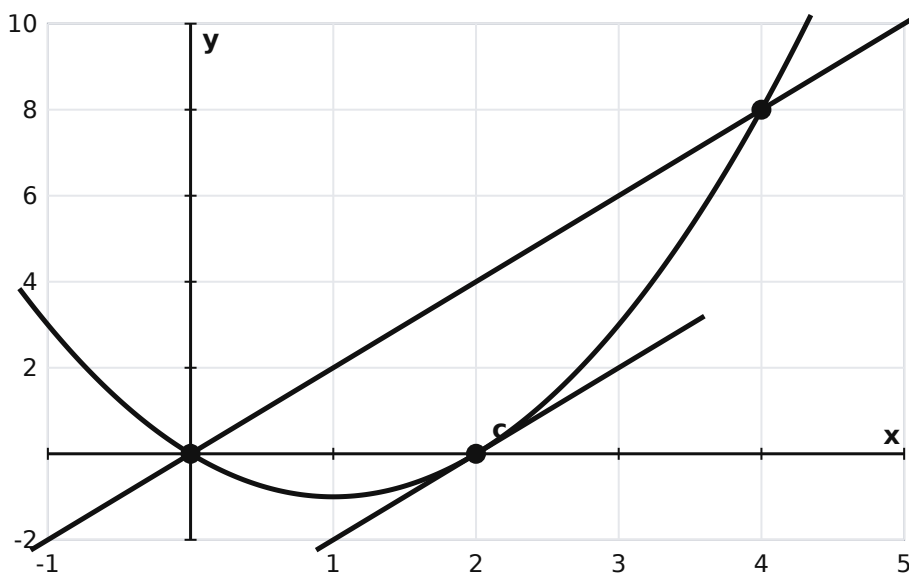
AP Calculus AB · Common Core State Standards: **AP CALC AB 5.1, AP CALC AB 5.2, AP CALC AB 5.3**

Question 1 - Verify the MVT [3 pts]

f is a polynomial, so it is continuous on $[0,4]$ and differentiable on $(0,4)$.

Average rate: $\frac{f(4)-f(0)}{4-0} = \frac{8-0}{4} = 2$. Solve $2c-2=2$: $c=2$.

Final answer: $c=2$.



Graph of $f(x) = x^2 - 2x$.

Scoring: 1 point for the hypotheses; 1 point for the average rate; 1 point for c .

Question 2 - Hypotheses fail [2 pts]

f is undefined at $x=2$, which is in the interval, so f is not continuous on $[1,3]$.

Scoring: 1 point for naming $x = 2$; 1 point for the conclusion about continuity.

Question 3 - MVT conclusion [2 pts]

g is differentiable, hence continuous. By the MVT there is a c with $g'(c) = \frac{-3-5}{5-1} = -2$.

Scoring: 1 point for the hypotheses; 1 point for the average rate -2.

Question 4 - Critical numbers [3 pts]

$f' = 4x^3 - 16x = 4x(x-2)(x+2)$: $x = -2, 0, 2$.

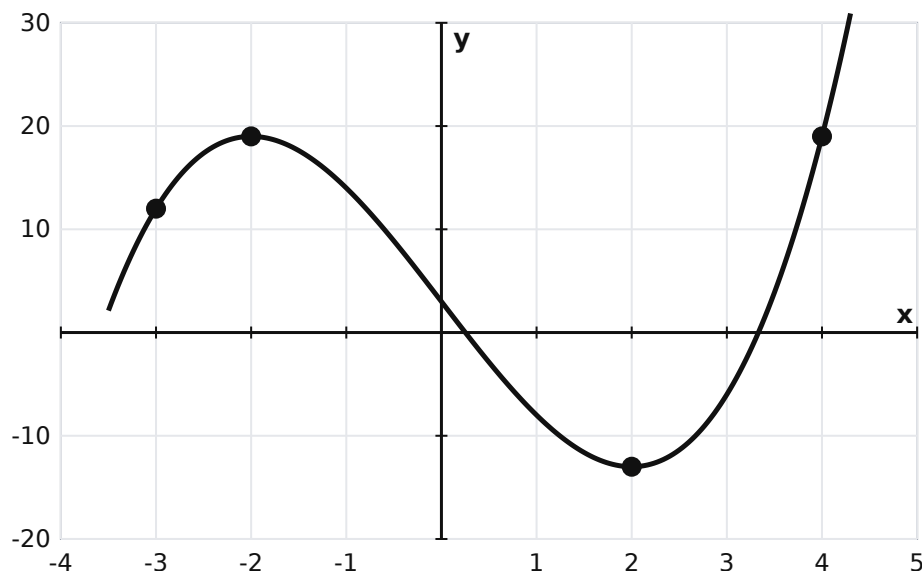
$h' = \frac{2}{3}(x-1)^{-1/3}$ is never zero but undefined at $x=1$.

Scoring: 1 point for f ; 1 point for the three numbers; 1 point for h .

Question 5 - Absolute extrema [3 pts]

$f' = 3x^2 - 12 = 0$ gives $x = \pm 2$. Candidates: $f(-3)=12$, $f(-2)=19$, $f(2)=-13$, $f(4)=19$.

Final answer: **absolute maximum 19 (at $x = -2$ and $x = 4$); absolute minimum -13 (at $x = 2$).**



Graph of $f(x) = x^3 - 12x + 3$.

Scoring: 1 point for critical numbers; 1 point for the four values; 1 point for the conclusions.

Question 6 - Speed and the MVT [2 pts]

Position is continuous and differentiable. The average speed is $\frac{150}{2.5} = 60$ mph, so by the MVT the instantaneous speed equals 60 mph at some time.

Scoring: 1 point for the average speed; 1 point for citing the MVT.

Question 7 - Extrema with an exponential [3 pts]

$f' = e^{-x}(1-x)$, zero at $x=1$. Candidates: $f(0)=0$, $f(1)=e^{-1} \approx 0.368$, $f(5)=5e^{-5} \approx 0.034$.

Final answer: **absolute maximum $1/e$ at $x = 1$; absolute minimum 0 at $x = 0$.**

Scoring: 1 point for f' and the critical number; 1 point for the values; 1 point for the conclusions.

Question 8 - Justifying a maximum [2 pts]

By the Extreme Value Theorem, h has an absolute maximum and minimum on $[1,6]$. They occur at the endpoints or the critical number.

The maximum is 9 at $x=3$; the minimum is 2 at $x=6$.

Scoring: 1 point for naming the EVT; 1 point for the two values with locations.

Total
20 points