

AP Calculus AB - Optimization - Lesson

Common Core AP CALC AB 5.10, AP CALC AB 5.11 · maths-pdf.fr/en/

Optimization finds the largest or smallest value of a quantity. Write the quantity to optimize, use the constraint to reduce it to one variable, find the critical numbers of the resulting function, and justify the extreme with the First Derivative Test, the Second Derivative Test or the candidates test. Then answer the question in a complete sentence with units.

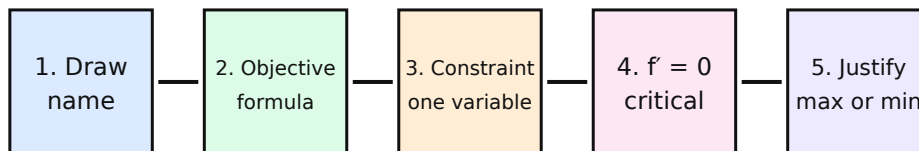
AP Calculus AB · Common Core State Standards: **AP CALC AB 5.10, AP CALC AB 5.11**

Key facts

- Name the variables, write an objective function, and use a constraint to reduce it to one variable.
- Find critical numbers by solving $f' = 0$ on the allowed domain.
- Justify the extreme with a sign change of f' , the sign of f'' , or by comparing endpoint values.
- Check the domain and give the final answer in context with units.

What is an optimization problem?

An optimization problem asks for the largest or smallest value of a quantity, such as the greatest area, the least cost or the highest profit. Calculus finds it by locating critical numbers of the quantity as a function of one variable.



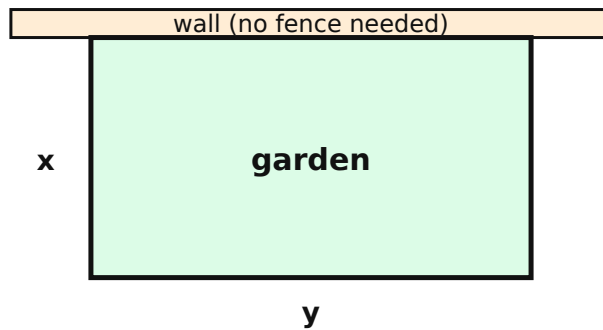
A five-step routine for optimization problems.

How do you set up and solve one?

1. Draw a picture and name the variables.
2. Write a formula for the *objective* (the quantity to maximize or minimize).
3. Use the *constraint* (a given perimeter, volume or cost) to rewrite the objective with one variable, and state the domain.
4. Differentiate, set the derivative equal to zero (or find where it is undefined), and solve.

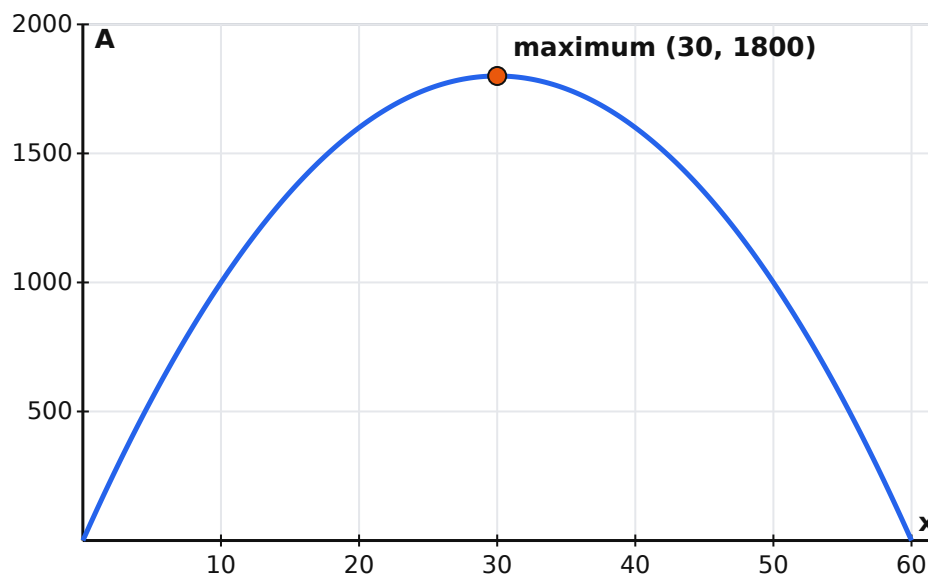
5. Justify that the answer is a maximum or minimum, and answer the question in a sentence with units.

Example (fence). A gardener has 120 ft of fence for three sides of a rectangle against a wall. With width x and length y , the constraint is $2x + y = 120$, so $y = 120 - 2x$ and the area is $A(x) = x(120 - 2x) = 120x - 2x^2$ for $0 < x < 60$.



Fence on three sides: $2x + y = 120$.

$A'(x) = 120 - 4x = 0$ gives $x = 30$, so $y = 60$ and $A = 1800$ square feet.



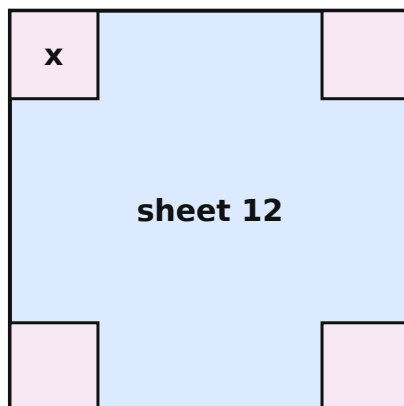
The area $A(x) = x(120 - 2x)$ is largest at $x = 30$.

How do you justify a maximum or minimum?

You must show that the critical number gives the extreme you claim. Three accepted methods: the **First Derivative Test** (the sign of A' goes from $+$ to $-$ at $x = 30$), the **Second Derivative Test** ($A'' = -4 < 0$, concave down), or the **candidates test** on a closed interval, comparing the critical value with the endpoint values.

A function with only one critical number on an interval, and with A' changing from positive to negative, has an *absolute* maximum there. On the open interval $(0,60)$ the area approaches 0 at both ends, so 1800 is the greatest.

What other models appear?



cut a square of side x from each corner, then fold up

The box has base $12 - 2x$ and height x .

For the open box, $V(x) = x(12-2x)^2$ on $0 < x < 6$. Then $V'(x) = 12(x-2)(x-6)$, so $x = 2$ gives the maximum volume $V(2) = 2 \cdot 64 = 128$ cubic inches. Always check that your answer is inside the domain and answers the question that was asked.

Questions about optimization

What is the first step in an optimization problem?

Draw a picture and name the variables. Then identify the quantity you want to maximize or minimize and the constraint that links the variables.

What is the difference between the objective and the constraint?

The objective is the formula you optimize, such as area. The constraint is the fixed condition, such as the amount of fence available, used to eliminate one variable.

How do you know a critical number gives a maximum?

Use the First Derivative Test (f' changes from positive to negative), the Second Derivative Test (f'' is negative), or compare values at the critical number and endpoints.

Why must you check the domain?

A critical number outside the allowed domain, such as a negative length, is not a valid answer.

Endpoints of the domain can also be the extreme values.

Why do the answers need units and a sentence?

The question is about a real situation, so the answer should say which quantity is optimal and give its value with units. A bare number does not answer the question.