

AP Calculus AB - Optimization - Answer Key

Common Core AP CALC AB 5.10, AP CALC AB 5.11 · maths-pdf.fr/en/

This answer key gives a full worked solution for every problem in the optimization worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 5.10, AP CALC AB 5.11**

Exercise 1 - Maximum product

Let the numbers be x and $20-x$. Then $P(x) = x(20-x) = 20x - x^2$ on $0 < x < 20$.

$P' = 20 - 2x = 0$ gives $x = 10$. $P'' = -2 < 0$, so it is a maximum.

Final answer: 10 and 10, product 100.

Exercise 2 - Minimum sum of squares

$S(x) = x^2 + (10-x)^2 = 2x^2 - 20x + 100$.

$S' = 4x - 20 = 0$ gives $x = 5$. $S'' = 4 > 0$: minimum.

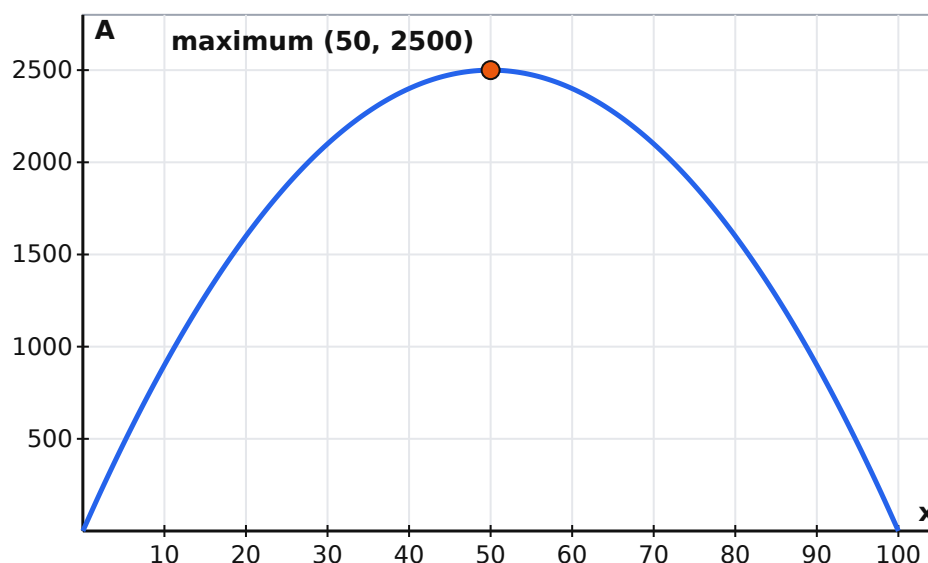
Final answer: 5 and 5, sum of squares 50.

Exercise 3 - Maximum area, fixed perimeter

Let the sides be x and $100-x$. $A = x(100-x)$.

$A' = 100 - 2x = 0$ gives $x = 50$, a 50×50 square. $A'' = -2 < 0$.

Final answer: 50 ft by 50 ft, area 2500 ft².



Area of a rectangle with perimeter 200 ft.

Exercise 4 - Fence against a wall

$2x + y = 120$ gives $y = 120 - 2x$. $A = x(120 - 2x)$ on $0 < x < 60$.

$A' = 120 - 4x = 0$ gives $x = 30$, $y = 60$. $A'' = -4 < 0$.

Final answer: 30 ft by 60 ft, area 1800 ft².

Exercise 5 - Minimum fence, fixed area

$xy = 800$, so $y = \frac{800}{x}$ and $P(x) = 2x + \frac{1600}{x}$ for $x > 0$.

$P' = 2 - \frac{1600}{x^2} = 0$ gives $x^2 = 800$, so $x = 20\sqrt{2} \approx 28.28$ and $y = 20\sqrt{2}$.

$P'' = \frac{3200}{x^3} > 0$: minimum.

Final answer: a square of side $20\sqrt{2} \approx 28.28$ ft; fence $80\sqrt{2} \approx 113.1$ ft.

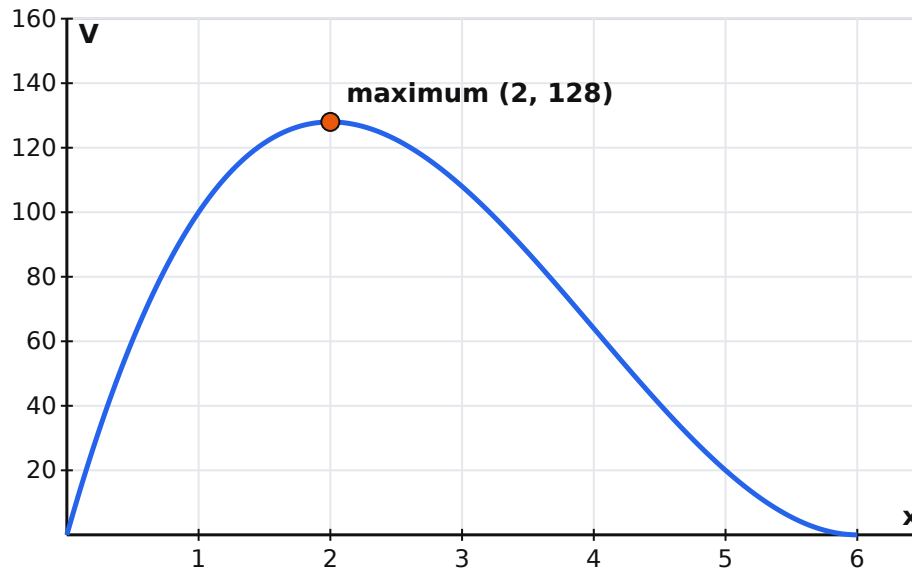
Exercise 6 - Open box from a square sheet

The base is $12 - 2x$ by $12 - 2x$ and the height is x : $V = x(12 - 2x)^2 = 4x^3 - 48x^2 + 144x$, $0 < x < 6$.

$V' = 12x^2 - 96x + 144 = 12(x - 2)(x - 6)$. Only $x = 2$ is in the domain.

$V''(2) = 24(2) - 96 = -48 < 0$: maximum. $V(2) = 2 \cdot 8^2 = 128$.

Final answer: $x=2$ in, maximum volume 128 in^3 .



Volume of the open box in cubic inches.

Exercise 7 - Open box from a rectangle

$$V = x(10-2x)(16-2x) = 4x^3 - 52x^2 + 160x \text{ for } 0 < x < 5.$$

$$V' = 12x^2 - 104x + 160 = 4(3x^2 - 26x + 40) = 4(3x-20)(x-2). \text{ In the domain: } x=2 \text{ (since } \frac{20}{3} > 5).$$

$$V(2) = 2 \cdot 6 \cdot 12 = 144.$$

Final answer: $x=2$ in, volume 144 in^3 .

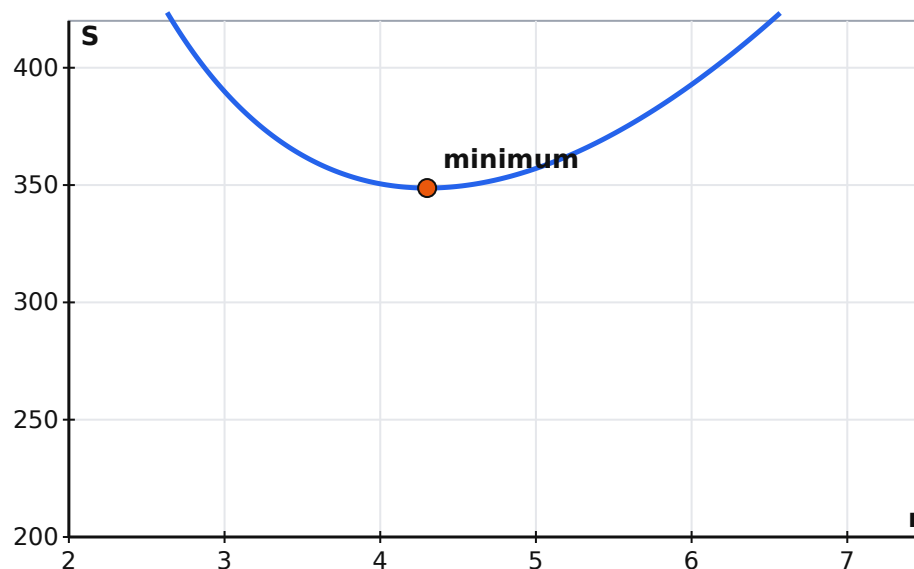
Exercise 8 - Can with least material

$$V = \pi r^2 h = 500 \text{ gives } h = \frac{500}{\pi r^2}. S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + \frac{1000}{r}.$$

$$S' = 4\pi r - \frac{1000}{r^2} = 0 \text{ gives } r^3 = \frac{250}{\pi}, \text{ so } r \approx 4.3 \text{ cm and } h = 2r \approx 8.6 \text{ cm.}$$

$$S'' = 4\pi + \frac{2000}{r^3} > 0: \text{ minimum.}$$

Final answer: $r \approx 4.3 \text{ cm}$, $h \approx 8.6 \text{ cm}$.



Surface area of the can in square centimeters.

Exercise 9 - Closest point on a parabola

Minimize the square of the distance: $D^2 = x^2 + (x^2 - 1)^2 = x^4 - x^2 + 1$.

$4x^3 - 2x = 2x(2x^2 - 1) = 0$: $x = 0$ or $x = \pm \frac{1}{\sqrt{2}}$. $D^2(0) = 1$ and $D^2\left(\pm \frac{1}{\sqrt{2}}\right) = \frac{3}{4}$.

Final answer: $\left(\pm \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$, distance $\frac{\sqrt{3}}{2}$.

Exercise 10 - Closest point on a line

$D^2 = x^2 + (2x + 1)^2 = 5x^2 + 4x + 1$. $(D^2)' = 10x + 4 = 0$ gives $x = -\frac{2}{5}$.

$y = 2\left(-\frac{2}{5}\right) + 1 = \frac{1}{5}$. $D^2 = \frac{1}{5}$.

Final answer: $\left(-\frac{2}{5}, \frac{1}{5}\right)$, distance $\frac{1}{\sqrt{5}}$.

Exercise 11 - Price for maximum revenue

$R(p) = p(500 - 10p) = 500p - 10p^2$. $R' = 500 - 20p = 0$ gives $p = 25$. $R'' = -20 < 0$.

$R(25) = 25(250) = 6250$.

Final answer: $p = \$25$, revenue \$6250 per week.

Exercise 12 - Maximizing profit

$$P = xp - C = x(20 - 0.01x) - (50 + 4x) = -0.01x^2 + 16x - 50.$$

$$P' = -0.02x + 16 = 0 \text{ gives } x = 800. \quad P'' = -0.02 < 0.$$

$$P(800) = -6400 + 12800 - 50 = 6350.$$

Final answer: 800 pies, profit \$6350.

Exercise 13 - Rectangle in a semicircle

With corners at $(\pm x, \sqrt{25 - x^2})$, $A = 2x\sqrt{25 - x^2}$ on $0 < x < 5$.

It is easier to maximize $A^2 = 4x^2(25 - x^2) = 100x^2 - 4x^4$: $200x - 16x^3 = 0$ gives $x^2 = \frac{25}{4}$, $x = \frac{5}{\sqrt{2}}$.

$$A = 2 \cdot \frac{5}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} = 25.$$

Final answer: 25 square units.

Exercise 14 - Rectangle under a parabola

$$A = 2x(12 - x^2) = 24x - 2x^3 \text{ for } 0 < x < \sqrt{12}.$$

$$A' = 24 - 6x^2 = 0 \text{ gives } x = 2. \quad A'' = -12x < 0.$$

$$A(2) = 2 \cdot 2 \cdot 8 = 32.$$

Final answer: 32 square units.

Exercise 15 - Fencing with two prices

Let the \$5 sides have length x and the \$10 sides length $y = \frac{800}{x}$. Cost:

$$C = 2(5x) + 2(10y) = 10x + \frac{16000}{x}.$$

$$C' = 10 - \frac{16000}{x^2} = 0 \text{ gives } x = 40, \quad y = 20. \quad C'' > 0: \text{ minimum.}$$

$$C(40) = 400 + 400 = 800.$$

Final answer: 40 ft by 20 ft, cost \$800.

Exercise 16 - Poster margins

Let the printed width be x , so its height is $\frac{200}{x}$. The paper is $(x+4)(\frac{200}{x}+8) = 232+8x+\frac{800}{x}$.

$8-\frac{800}{x^2}=0$ gives $x=10$. Printed area: 10×20 . Paper: $14 \times 28 = 392$.

Final answer: 14 cm by 28 cm, area 392 cm².

Exercise 17 - Open-top box

$x^2h=1000$ gives $h=\frac{1000}{x^2}$. $S=x^2+4xh=x^2+\frac{4000}{x}$.

$S'=2x-\frac{4000}{x^2}=0$ gives $x^3=2000$, $x \approx 12.6$ in and $h=\frac{1000}{x^2} \approx 6.3$ in.

$S''=2+\frac{8000}{x^3}>0$: minimum.

Final answer: $x \approx 12.6$ in, $h \approx 6.3$ in.

Exercise 18 - Closest point to (4, 0)

$D^2=(x-4)^2+x$. $(D^2)'=2(x-4)+1=0$ gives $x=\frac{7}{2}$.

$D^2=\frac{1}{4}+\frac{7}{2}=\frac{15}{4}$, so $D=\frac{\sqrt{15}}{2} \approx 1.94$.

Final answer: $(\frac{7}{2}, \sqrt{\frac{7}{2}})$.

Exercise 19 - Two adjacent pens

The fence has three sides of length x (two outside and one middle) and two sides of length y :
 $3x+2y=1000$, so $y=500-\frac{3x}{2}$.

$A=xy=500x-\frac{3}{2}x^2$, $A'=500-3x=0$ gives $x=\frac{500}{3} \approx 166.7$ and $y=250$.

$A=\frac{500}{3} \cdot 250 \approx 41,667$ square feet.

Final answer: $x=\frac{500}{3}$ ft, $y=250$ ft, $A \approx 41,667$ ft².

Exercise 20 - Ticket price

With x dollars of increase: $R(x)=(12+x)(200-10x)=2400+80x-10x^2$.

$R' = 80 - 20x = 0$ gives $x = 4$. $R'' = -20 < 0$. Price $12 + 4 = 16$; $R(4) = 16 \cdot 160 = 2560$.

Final answer: \$16 per ticket, revenue \$2560.