

AP Calculus AB - Optimization - Test Answer Key

Common Core AP CALC AB 5.10, AP CALC AB 5.11 · maths-pdf.fr/en/

Use this answer key to score the optimization test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

AP Calculus AB · Common Core State Standards: **AP CALC AB 5.10, AP CALC AB 5.11**

Question 1 - Maximum product [2 pts]

$P = x(30 - x)$, $P' = 30 - 2x = 0$ gives $x = 15$. $P'' = -2 < 0$. Both numbers are 15 (product 225).

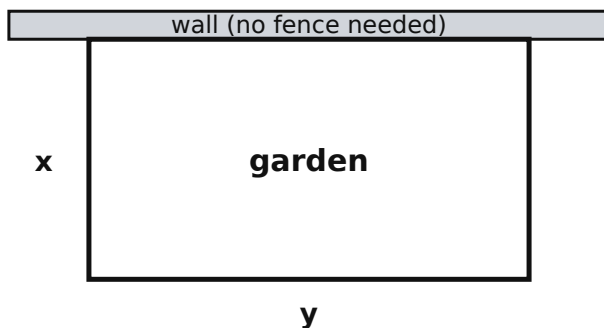
Scoring: 1 point for the function and derivative; 1 point for the answer with justification.

Question 2 - Pen against a wall [3 pts]

$2x + y = 80$, $y = 80 - 2x$. $A = x(80 - 2x) = 80x - 2x^2$, $0 < x < 40$.

$A' = 80 - 4x = 0$ gives $x = 20$, $y = 40$. $A'' = -4 < 0$.

Final answer: **20 ft by 40 ft; maximum area 800 ft².**



Pen against a wall; 80 ft of fence.

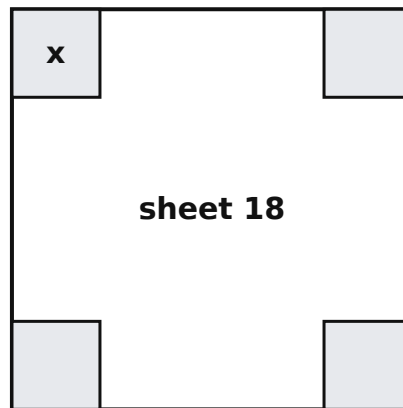
Scoring: 1 point for the area function with domain; 1 point for the critical number; 1 point for the justification and final answer.

Question 3 - Open box [3 pts]

$$V = x(18-2x)^2, 0 < x < 9. V' = (18-2x)^2 - 4x(18-2x) = (18-2x)(18-6x).$$

Critical numbers: $x=9$ (endpoint) and $x=3$. V' goes from $+$ to $-$ at $x=3$. $V(3) = 3 \cdot 144 = 432$.

Final answer: $x=3$ in, maximum volume **432 in³**.



cut a square of side x from each corner, then fold up

An 18-inch square sheet.

Scoring: 1 point for $V(x)$; 1 point for the derivative and critical number; 1 point for justification and the volume.

Question 4 - Closest point [2 pts]

$$D^2 = x^2 + (x+3)^2 = 2x^2 + 6x + 9. 4x + 6 = 0 \text{ gives } x = -\frac{3}{2}, y = \frac{3}{2}.$$

Final answer: $\left(-\frac{3}{2}, \frac{3}{2}\right)$.

Scoring: 1 point for the distance function; 1 point for the point.

Question 5 - Revenue [3 pts]

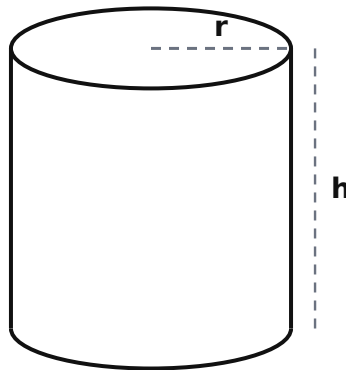
- $R(p) = p(300 - 6p) = 300p - 6p^2$.
- $R' = 300 - 12p = 0$ gives $p = 25$.
- $R'' = -12 < 0$, a maximum; $R(25) = 25 \cdot 150 = 3750$ dollars per week.

Scoring: 1 point per part.

Question 6 - Can design [3 pts]

$$h = \frac{1000}{\pi r^2}. S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + \frac{2000}{r}.$$

$$S' = 4\pi r - \frac{2000}{r^2} = 0 \text{ gives } r^3 = \frac{500}{\pi}, r \approx 5.42 \text{ cm. } S'' = 4\pi + \frac{4000}{r^3} > 0: \text{ minimum.}$$



A can holding 1000 cm^3 .

Scoring: 1 point for $S(r)$; 1 point for the critical number; 1 point for the justification.

Question 7 - Justifying an extremum [2 pts]

$$A' = 60 - 4x = 0 \text{ gives } x = 15. A' > 0 \text{ for } 0 < x < 15 \text{ and } A' < 0 \text{ for } 15 < x < 30.$$

So A increases then decreases: absolute maximum $A(15) = 15 \cdot 30 = 450$.

Scoring: 1 point for the sign analysis; 1 point for the conclusion and value.

Question 8 - Ticket price [2 pts]

With n increases of \\$2: price $20 + 2n$ and tickets $400 - 20n$.

$$R(n) = (20 + 2n)(400 - 20n) = 8000 + 400n - 40n^2.$$

$$R' = 400 - 80n = 0 \text{ gives } n = 5. \text{ Price } 20 + 10 = 30, \text{ revenue } 30 \cdot 300 = 9000.$$

Final answer: **\\$30 per ticket, revenue \\$9000.**

Scoring: 1 point for $R(n)$; 1 point for the answer.

Total

20 points