

AP Calculus AB - Product Rule, Quotient Rule and Trig Derivatives - Lesson

Common Core AP CALC AB 2.8, AP CALC AB 2.9, AP CALC AB 2.10 · maths-pdf.fr/en/

The product rule says $(uv)' = u'v + uv'$, and the quotient rule says $(u/v)' = (u'v - uv')/v^2$. Never differentiate a product or quotient factor by factor. The derivatives of tan, cot, sec and csc are $\sec^2x$, $-\csc^2x$, $\sec x \tan x$ and $-\csc x \cot x$.

AP Calculus AB · Common Core State Standards: **AP CALC AB 2.8, AP CALC AB 2.9, AP CALC AB 2.10**

Key facts

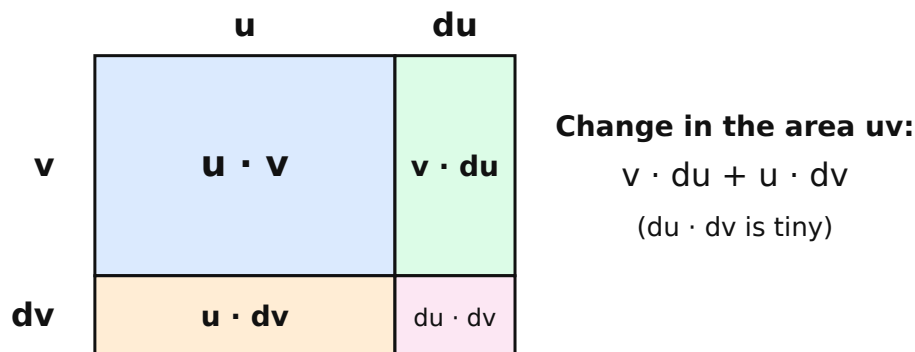
- Product rule: $(uv)' = u'v + uv'$.
- Quotient rule: $(u/v)' = (u'v - uv')/v^2$, and the order in the numerator matters.
- $d/dx \tan x = \sec^2x$ and $d/dx \sec x = \sec x \tan x$.
- The derivatives of $\cot x$ and $\csc x$ are negative: $-\csc^2x$ and $-\csc x \cot x$.

What is the product rule?

The derivative of a product is not the product of the derivatives. To differentiate $f(x) = u(x) v(x)$, each factor takes a turn being differentiated.

Product rule

$\frac{d}{dx}[uv] = u'v + uv'$. In words: derivative of the first times the second, plus the first times the derivative of the second.



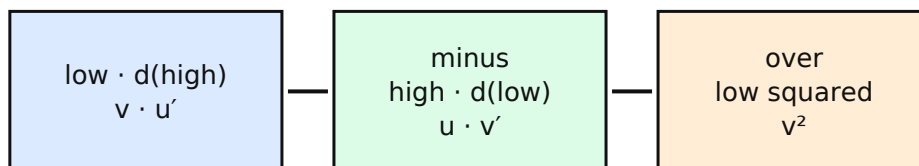
A rectangle with sides u and v grows by $v \cdot du + u \cdot dv$; the corner $du \cdot dv$ is negligible.

Example. For $f(x) = x^2 e^x$, take $u = x^2$ and $v = e^x$. Then $f'(x) = 2x e^x + x^2 e^x = e^x(x^2 + 2x)$.

What is the quotient rule?

Quotient rule

$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - uv'}{v^2}$, where $v \neq 0$. The order in the numerator matters because of the subtraction.



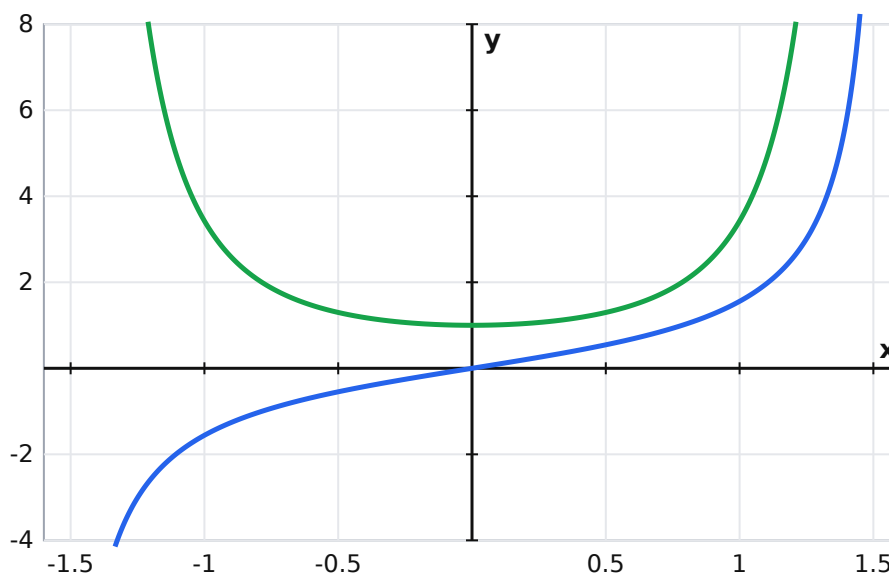
The quotient rule: $(u/v)' = (v \cdot u' - u \cdot v') / v^2$.

Example. For $f(x) = \frac{x^2+1}{x-3}$: $f'(x) = \frac{2x(x-3) - (x^2+1)(1)}{(x-3)^2} = \frac{x^2-6x-1}{(x-3)^2}$.

What are the derivatives of the other trig functions?

Function	Derivative
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$

The quotient rule proves the first row: $\tan x = \frac{\sin x}{\cos x}$ gives $\frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \sec^2 x$. The three "co" functions all carry a negative sign.

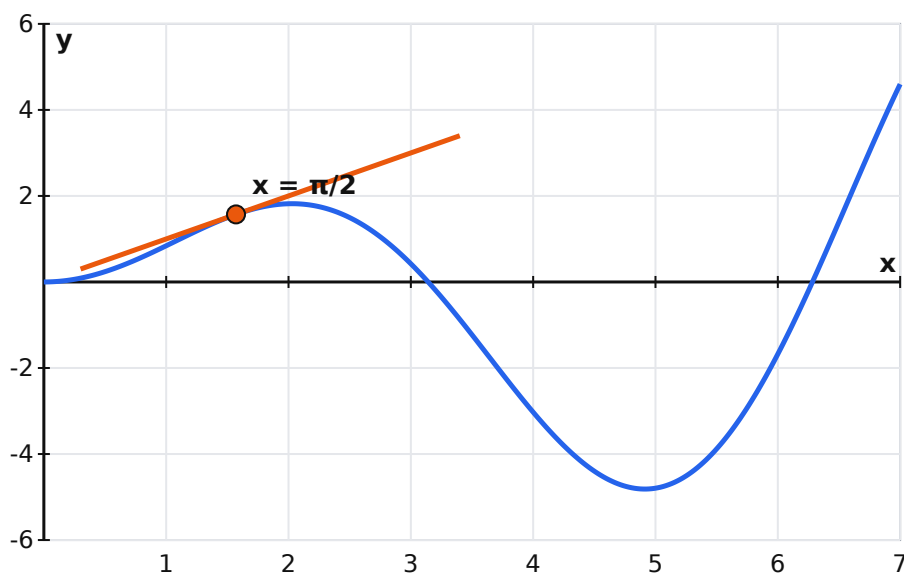


$\tan x$ (blue) and its derivative $\sec^2 x$ (green). The slope of $\tan x$ is never below 1.

How do you use the rules together?

1. Decide whether the expression is a product, a quotient or a sum of those.
2. Name the pieces u and v and write both derivatives.
3. Substitute into the rule and simplify, factoring common terms when possible.
4. Evaluate at the given x if a slope is needed.

For $f(x) = x \sin x$, $f'(x) = \sin x + x \cos x$, so the slope at $x = \frac{\pi}{2}$ is $1 + 0 = 1$.



For $f(x) = x \sin x$, the slope at $x = \pi/2$ is $f'(\pi/2) = 1$.

What mistakes should you avoid?

- Writing $(uv)' = u'v'$. The product rule always has two terms.
- Reversing the quotient rule numerator. Write the subtraction as $u'v - uv'$, never the other way around.
- Forgetting to square the denominator v in the quotient rule.
- Using a quotient rule when you can simplify first: $\frac{x^3+x}{x} = x^2+1$ differentiates to $2x$ in one step.

When a function mixes rules, work from the outside in: identify the main operation first (sum, product or quotient) and apply that rule before you differentiate the inner pieces.

Questions about product rule, quotient rule and trig derivatives

What is the product rule?

The product rule states that the derivative of u times v equals u -prime times v plus u times v -prime. Each factor gets differentiated once while the other stays unchanged.

Is the derivative of a product the product of the derivatives?

No. For x times x , which is x squared, the derivative is $2x$, but the product of the derivatives is 1 times 1 , which is 1 . Use the product rule instead.

How do you remember the quotient rule?

Say "low d-high minus high d-low, over low squared." The numerator is v times u -prime minus u times v -prime, and the denominator is v squared.

What is the derivative of $\tan x$?

The derivative of $\tan x$ is $\sec^2 x$. You can prove it by writing $\tan x$ as $\sin x$ over $\cos x$ and applying the quotient rule.

When should I simplify before differentiating?

Simplify first when a quotient divides evenly, such as $(x^3 + x)/x = x^2 + 1$. It avoids the quotient rule and reduces algebra mistakes.