

AP Calculus AB - Product Rule, Quotient Rule and Trig Derivatives - Answer Key

Common Core AP CALC AB 2.8, AP CALC AB 2.9, AP CALC AB 2.10 · maths-pdf.fr/en/

This answer key gives a full worked solution for every problem in the product rule worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 2.8, AP CALC AB 2.9, AP CALC AB 2.10**

Exercise 1 - Product with an exponential

Let $u = x^2$ and $v = e^x$. Then $u' = 2x$ and $v' = e^x$.

Product rule: $(uv)' = u'v + uv' = (2x)(e^x) + (x^2)(e^x)$.

Final answer: $f'(x) = (x^2 + 2x)e^x$.

Exercise 2 - Product of two polynomials

Let $u = 3x + 1$ and $v = x^2 - 4$. Then $u' = 3$ and $v' = 2x$.

Product rule: $(uv)' = u'v + uv' = (3)(x^2 - 4) + (3x + 1)(2x)$.

Final answer: $f'(x) = 9x^2 + 2x - 12$.

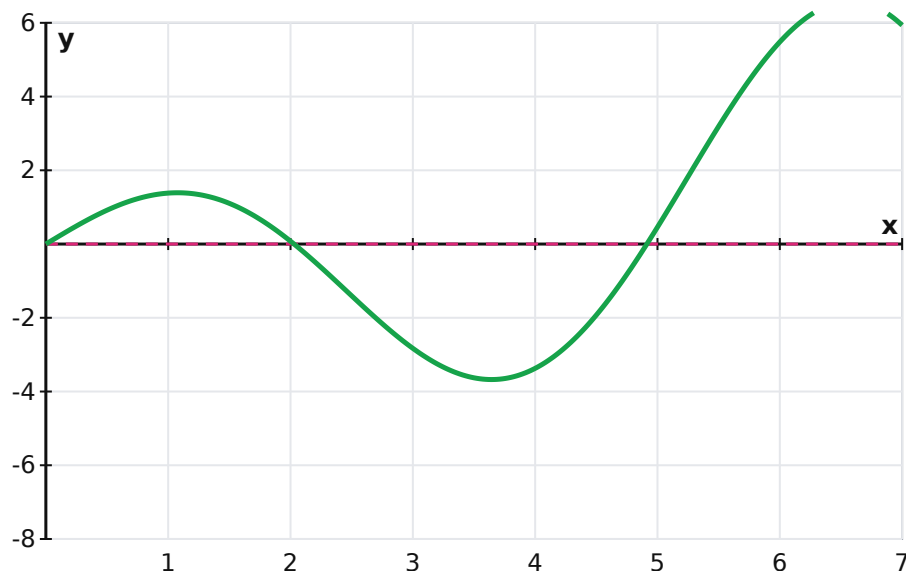
Check: $(3x + 1)(x^2 - 4) = 3x^3 + x^2 - 12x - 4$, whose derivative is $9x^2 + 2x - 12$. The answers match.

Exercise 3 - Product with sine

Let $u = x$ and $v = \sin(x)$. Then $u' = 1$ and $v' = \cos(x)$.

Product rule: $(uv)' = u'v + uv' = (1)(\sin(x)) + (x)(\cos(x))$.

Final answer: $f'(x) = x\cos(x) + \sin(x)$.



The derivative $f'(x) = \sin x + x \cos x$.

Exercise 4 - Exponential times log

Let $u = e^x$ and $v = \ln(x)$. Then $u' = e^x$ and $v' = \frac{1}{x}$.

Product rule: $(uv)' = u'v + uv' = (e^x)(\ln(x)) + (e^x)(\frac{1}{x})$.

Final answer: $f'(x) = e^x \ln(x) + \frac{e^x}{x}$.

Exercise 5 - Square root times cosine

Let $u = \sqrt{x}$ and $v = \cos(x)$. Then $u' = \frac{1}{2\sqrt{x}}$ and $v' = -\sin(x)$.

Product rule: $(uv)' = u'v + uv' = (\frac{1}{2\sqrt{x}})(\cos(x)) + (\sqrt{x})(-\sin(x))$.

Final answer: $f'(x) = -\sqrt{x}\sin(x) + \frac{\cos(x)}{2\sqrt{x}}$.

Exercise 6 - Simple quotient

Let $u = x$ and $v = x+1$. Then $u' = 1$ and $v' = 1$.

Quotient rule: $(\frac{u}{v})' = \frac{u'v - uv'}{v^2} = \frac{(1)(x+1) - (x)(1)}{(x+1)^2}$.

Final answer: $f'(x) = \frac{1}{(x+1)^2}$.

Exercise 7 - Quotient with a quadratic

Let $u = x^2 + 1$ and $v = x - 3$. Then $u' = 2x$ and $v' = 1$.

$$\text{Quotient rule: } \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} = \frac{(2x)(x-3) - (x^2+1)(1)}{(x-3)^2}.$$

$$\text{Final answer: } f'(x) = \frac{x^2 - 6x - 1}{(x-3)^2}.$$

Exercise 8 - Cosine over a power

Let $u = \cos x$ and $v = x^2$. Then $u' = -\sin x$ and $v' = 2x$.

$$\text{Quotient rule: } \frac{-\sin x \cdot x^2 - \cos x \cdot 2x}{x^4} = \frac{-x \sin x - 2 \cos x}{x^3}.$$

$$\text{Final answer: } f'(x) = -\frac{x \sin x + 2 \cos x}{x^3}.$$

Exercise 9 - Sine over x

Let $u = \sin(x)$ and $v = x$. Then $u' = \cos(x)$ and $v' = 1$.

$$\text{Quotient rule: } \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} = \frac{(\cos(x))(x) - (\sin(x))(1)}{(x)^2}.$$

$$\text{Final answer: } f'(x) = \frac{x \cos(x) - \sin(x)}{x^2}.$$

Exercise 10 - Log over a power

Let $u = \ln(x)$ and $v = x^3$. Then $u' = \frac{1}{x}$ and $v' = 3x^2$.

$$\text{Quotient rule: } \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} = \frac{\left(\frac{1}{x}\right)(x^3) - (\ln(x))(3x^2)}{(x^3)^2}.$$

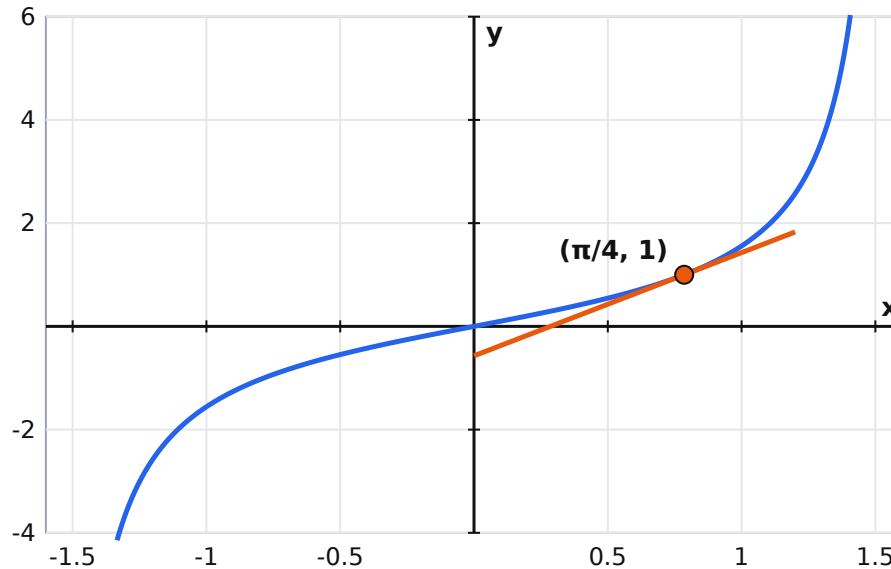
$$\text{Final answer: } f'(x) = \frac{1 - 3 \ln(x)}{x^4}.$$

Exercise 11 - Derivative of tangent

$$\frac{d}{dx}[\tan x] = \sec^2 x.$$

$$\text{At } x = \frac{\pi}{4}, \sec \frac{\pi}{4} = \sqrt{2}, \text{ so } \sec^2 \frac{\pi}{4} = 2.$$

Final answer: $\sec^2 x$ and the value 2.



The tangent to $\tan x$ at $x = \pi/4$ has slope 2.

Exercise 12 - Product with tangent

Let $u = x$ and $v = \tan(x)$. Then $u' = 1$ and $v' = \tan^2(x) + 1$.

Product rule: $(uv)' = u'v + uv' = (1)(\tan(x)) + (x)(\tan^2(x) + 1)$.

Final answer: $f'(x) = x\sec^2(x) + \tan(x)$.

Exercise 13 - Secant at a point

$\frac{d}{dx}[\sec x] = \sec x \tan x$. At $\frac{\pi}{3}$: $\sec \frac{\pi}{3} = 2$ and $\tan \frac{\pi}{3} = \sqrt{3}$.

Final answer: $2\sqrt{3}$.

Exercise 14 - Cotangent and cosecant

$f'(x) = -\csc^2 x - \csc x \cot x$.

Final answer: $-\csc x(\csc x + \cot x)$.

Exercise 15 - Tangent line with a product

$$f(\pi) = \pi^2 \sin \pi = 0. \quad f'(x) = 2x \sin x + x^2 \cos x, \text{ so } f'(\pi) = 0 + \pi^2(-1) = -\pi^2.$$

$$\text{Tangent: } y - 0 = -\pi^2(x - \pi). \text{ Final answer: } y = -\pi^2(x - \pi).$$

Exercise 16 - Horizontal tangent

$$f'(x) = e^x + x e^x = e^x(1+x). \text{ Since } e^x > 0, \text{ set } 1+x = 0.$$

$$\text{Final answer: } x = -1, \text{ the point } \left(-1, -\frac{1}{e}\right).$$

Exercise 17 - Quotient and a horizontal tangent

$$\text{Let } u = x^2 \text{ and } v = x^2 + 1. \text{ Then } u' = 2x \text{ and } v' = 2x.$$

$$\text{Quotient rule: } \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} = \frac{(2x)(x^2+1) - (x^2)(2x)}{(x^2+1)^2}.$$

$$\text{Final answer: } f'(x) = \frac{2x}{(x^2+1)^2}.$$

The numerator is zero only at $x = 0$, so the only horizontal tangent is at $(0,0)$.

Exercise 18 - Using a table of values

$$\text{Product: } f'g + fg' = (-1)(4) + (3)(5) = 11.$$

$$\text{Quotient: } \frac{f'g - fg'}{g^2} = \frac{(-1)(4) - (3)(5)}{16} = -\frac{19}{16}.$$

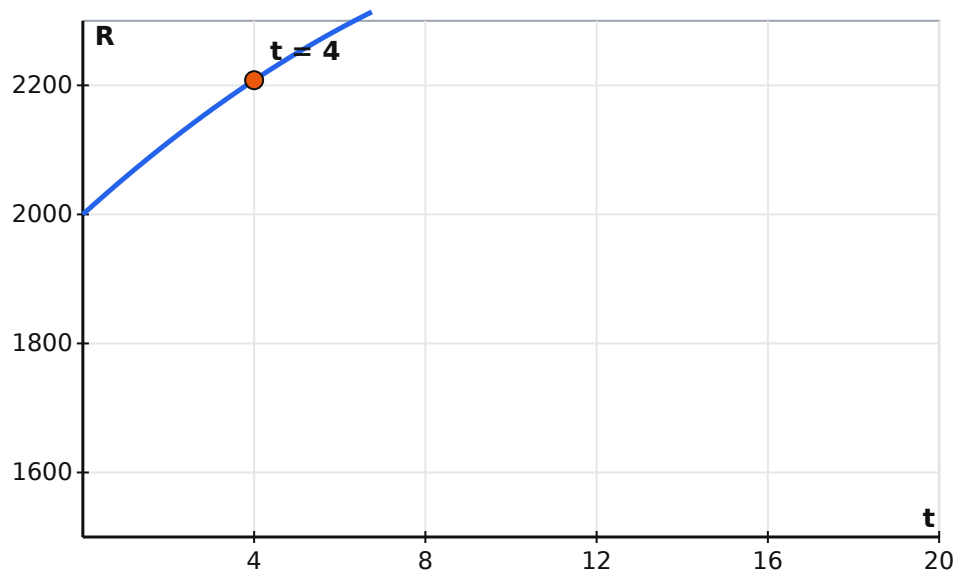
$$\text{Final answer: } 11 \text{ and } -\frac{19}{16}.$$

Exercise 19 - Revenue growth

$$p(4) = 12, \quad q(4) = 184, \quad p' = 0.5, \quad q' = -4.$$

$$\text{Product rule: } R'(4) = p'q + pq' = 0.5(184) + 12(-4) = 92 - 48 = 44.$$

Final answer: **\\$44 per week**: at 4 weeks the revenue is growing by about \\$44 each week.



Revenue in dollars during the first 20 weeks.

Exercise 20 - Peak concentration

Let $u = x$ and $v = x^2 + 4$. Then $u' = 1$ and $v' = 2x$.

Quotient rule: $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} = \frac{(1)(x^2 + 4) - (x)(2x)}{(x^2 + 4)^2}$.

Final answer: $f'(x) = \frac{4 - x^2}{(x^2 + 4)^2}$.

Set $4 - x^2 = 0$: $x = 2$ (hours cannot be negative).

$C(2) = \frac{2}{8} = 0.25$. The concentration is greatest at **2 hours** (0.25 mg/L).

Exercise 21 - Three factors

Apply the product rule twice. Group $x^2 e^x$ as the first factor, whose derivative is $e^x(x^2 + 2x)$.

$f'(x) = e^x(x^2 + 2x)\sin x + x^2 e^x \cos x$.

Final answer: $x e^x [(x + 2)\sin x + x \cos x]$.

Exercise 22 - Proving the tangent derivative

$u = \sin x$, $v = \cos x$, $u' = \cos x$, $v' = -\sin x$.

$$\frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x.$$

Final answer: $(\tan x)' = \sec^2 x$.