

AP Calculus AB - Product Rule, Quotient Rule and Trig Derivatives - Test Answer Key

Common Core AP CALC AB 2.8, AP CALC AB 2.9, AP CALC AB 2.10 · maths-pdf.fr/en/

Use this answer key to score the product rule test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

AP Calculus AB · Common Core State Standards: **AP CALC AB 2.8, AP CALC AB 2.9, AP CALC AB 2.10**

Question 1 - Product rule [2 pts]

$$f'(x) = 2x e^x + (x^2 + 3)e^x = e^x(x^2 + 2x + 3).$$

Final answer: $e^x(x^2 + 2x + 3)$.

Scoring: 1 point for the product rule setup; 1 point for the simplified result.

Question 2 - Quotient rule [3 pts]

$$f'(x) = \frac{2(x+4) - (2x-1)(1)}{(x+4)^2} = \frac{9}{(x+4)^2}.$$

Final answer: $\frac{9}{(x+4)^2}$.

Scoring: 1 point for u' and v' ; 1 point for the quotient rule setup; 1 point for simplifying.

Question 3 - Trig and product [2 pts]

$$g'(x) = \cos x - x \sin x + 3 \sec^2 x.$$

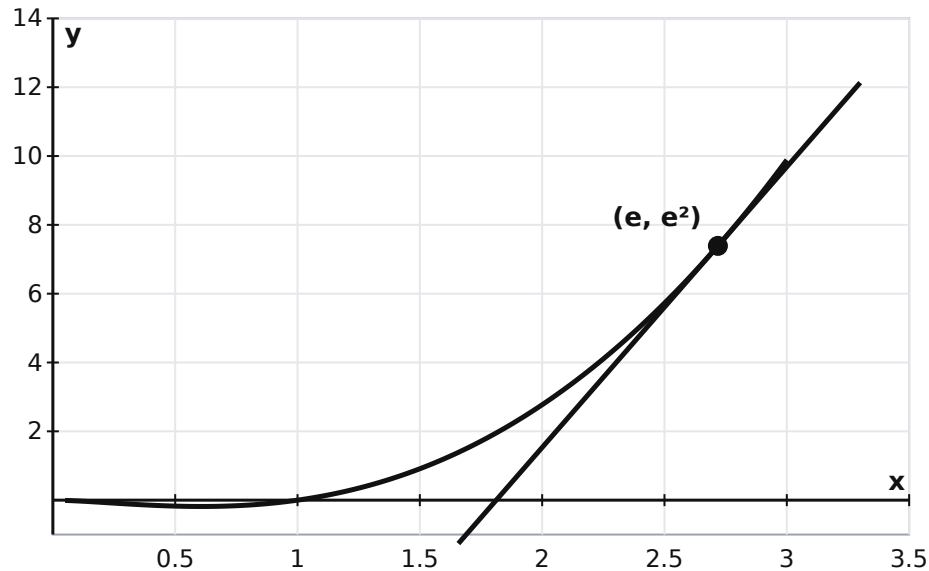
Final answer: $\cos x - x \sin x + 3 \sec^2 x$.

Scoring: 1 point for the product term; 1 point for the tangent term.

Question 4 - Tangent line [3 pts]

$$f(e) = e^2 \ln e = e^2. \quad f'(x) = 2x \ln x + x, \text{ so } f'(e) = 2e + e = 3e.$$

$$\text{Tangent: } y - e^2 = 3e(x - e). \text{ Final answer: } y = 3ex - 2e^2.$$



Graph of $f(x) = x^2 \ln x$.

Scoring: 1 point for $f(e)$; 1 point for $f'(e)$; 1 point for the equation.

Question 5 - Table of values [2 pts]

$$(fg)'(1) = 3(-1) + 2(4) = 5.$$

$$\left(\frac{f}{g}\right)'(1) = \frac{3(-1) - 2(4)}{(-1)^2} = -11.$$

Final answer: 5 and -11 .

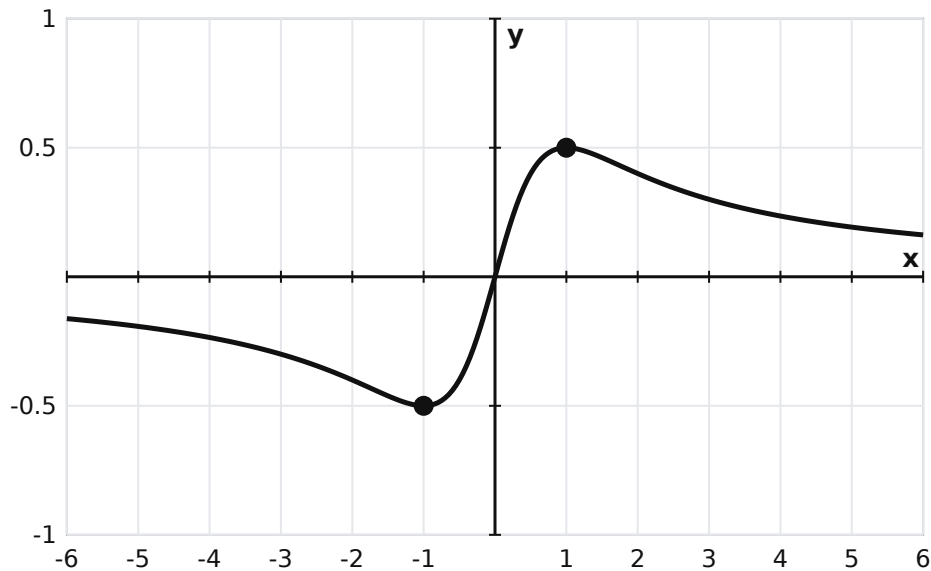
Scoring: 1 point for each derivative.

Question 6 - Horizontal tangents [3 pts]

$$h'(x) = \frac{1(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}. \text{ Set } 1-x^2=0: x = \pm 1.$$

$$h(1) = \frac{1}{2} \text{ and } h(-1) = -\frac{1}{2}.$$

Final answer: $\left(1, \frac{1}{2}\right)$ and $\left(-1, -\frac{1}{2}\right)$.



Graph of $h(x) = x/(x^2 + 1)$.

Scoring: 1 point for h' ; 1 point for the x -values; 1 point for the points.

Question 7 - Rate of sales [3 pts]

a. $R(t) = (2 + 0.1t)(120 - 2t)$.

b. $R'(t) = 0.1(120 - 2t) + (2 + 0.1t)(-2)$, so $R'(10) = 0.1(100) + 3(-2) = 10 - 6 = 4$ dollars per day.

Scoring: 1 point for (a); 1 point for the product rule setup; 1 point for the value 4 with units.

Question 8 - Secant and cosecant [2 pts]

$$k'(x) = \sec x \tan x + 2 \csc x \cot x.$$

At $\frac{\pi}{4}$: $\sqrt{2} \cdot 1 + 2 \cdot \sqrt{2} \cdot 1 = 3\sqrt{2}$.

Final answer: $3\sqrt{2}$.

Scoring: 1 point for the derivative; 1 point for the value.

Total

20 points