

AP Calculus AB - Rates of Change in Context and Straight-Line Motion - Lesson

Common Core AP CALC AB 4.1, AP CALC AB 4.2, AP CALC AB 4.3 · maths-pdf.fr/en/

In context, a derivative is a rate of change measured in output units per input unit. For a particle with position $s(t)$, velocity is $v = s'$, acceleration is $a = v'$, and speed is $|v|$. The particle moves right when v is positive, left when v is negative, and speeds up when v and a have the same sign.

AP Calculus AB · Common Core State Standards: **AP CALC AB 4.1, AP CALC AB 4.2, AP CALC AB 4.3**

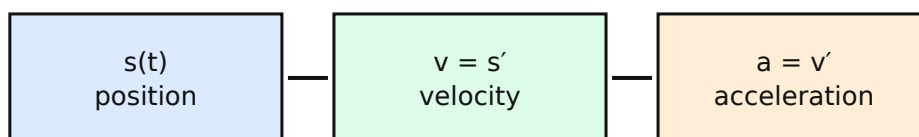
Key facts

- A derivative has units of output per input, such as dollars per pencil or feet per second.
- Velocity is $s'(t)$, acceleration is $v'(t)$, and speed is the absolute value of velocity.
- Speed increases when velocity and acceleration have the same sign.
- Total distance adds the distances between turning points, where velocity changes sign.

What does a derivative mean in context?

A derivative is a rate of change, and its units are **units of output per unit of input**. If $C(x)$ is the cost in dollars of x pencils, then $C'(500) = 0.12$ means that at 500 pencils the cost is increasing at about 0.12 dollars per pencil. A complete interpretation names the quantity, the input value, the units, and whether it is increasing or decreasing.

How do position, velocity and acceleration connect?

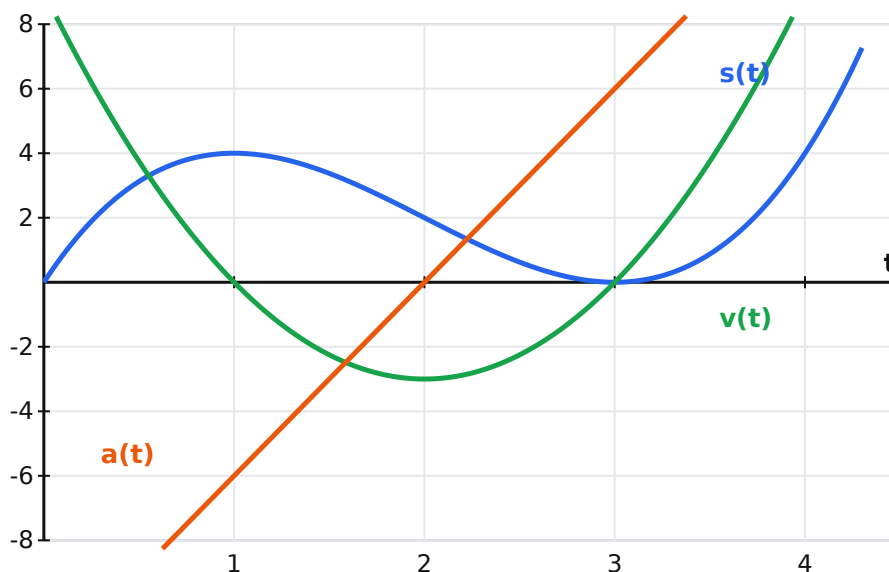


Differentiate position to get velocity, and velocity to get acceleration.

Straight-line motion

- $v(t) = s'(t)$ is velocity; $a(t) = v'(t) = s''(t)$ is acceleration.

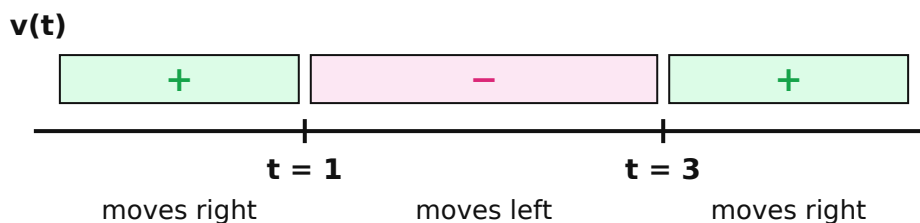
- Speed is $|v(t)|$, which is never negative.
- The particle is at rest when $v(t) = 0$.
- It moves right (positive direction) when $v > 0$ and left when $v < 0$.



Position (blue), velocity (green) and acceleration (orange) of one particle.

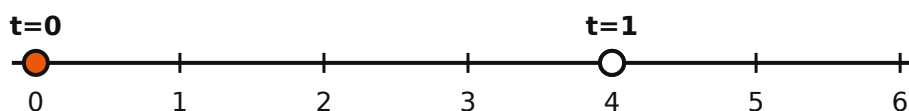
When is a particle speeding up or slowing down?

Compare the signs of v and a . If they have the *same* sign, the speed is increasing. If they have *opposite* signs, the speed is decreasing. For $s(t) = t^3 - 6t^2 + 9t$, $v = 3(t-1)(t-3)$ and $a = 6t - 12$. At $t = 0.5$, $v > 0$ and $a < 0$: the particle is slowing down while moving right.



Velocity signs show the direction of motion.

The particle changes direction only where v changes sign. Here that happens at $t = 1$ and $t = 3$, which you can read from the sign chart.



The particle runs right to 4, back to 0, then right again.

Total distance is found by adding the distances traveled on each stretch between turning points. From $t=0$ to $t=4$, $s(0)=0$, $s(1)=4$, $s(3)=0$ and $s(4)=4$, so the distance is $4+4+4=12$.

Displacement and distance are different. Displacement is the net change in position, $s(b)-s(a)$, and it can be zero. Distance is always positive and counts every stretch of the trip, including backward motion.

What other rates of change appear in problems?

Any rate has the same structure. For a circle, $A(r)=\pi r^2$ gives $A'(r)=2\pi r$, the rate at which area grows per unit of radius. At $r=5$ inches the area grows at 10π square inches per inch of radius.

Questions about rates of change in context and straight-line motion

How do you interpret a derivative in context?

State the quantity, the input value, the numerical rate with units, and the direction of change. For example, $T'(3) = -4.5$ means the temperature is falling at 4.5 degrees Fahrenheit per hour at 3 hours.

What is the difference between velocity and speed?

Velocity has a direction and can be negative. Speed is the absolute value of velocity, so it is never negative.

When is a particle speeding up?

It is speeding up when velocity and acceleration have the same sign, and slowing down when they have opposite signs.

How do you find when a particle changes direction?

Find where $v(t) = 0$ and check that the sign of v changes there. A zero of velocity without a sign change is a pause, not a turn.

How do you find total distance traveled?

Find the turning points, evaluate the position at the endpoints and turning points, and add the absolute changes between consecutive positions.