

AP Calculus AB - Rates of Change in Context and Straight-Line Motion - Answer Key

Common Core AP CALC AB 4.1, AP CALC AB 4.2, AP CALC AB 4.3 · maths-pdf.fr/en/

This answer key gives a full worked solution for every problem in the rates of change worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 4.1, AP CALC AB 4.2, AP CALC AB 4.3**

Exercise 1 - Interpreting a cost derivative

The units of C' are dollars per flyer.

Final answer: **When 500 flyers are printed, the cost is increasing at about \$0.12 per additional flyer.**

Exercise 2 - Interpreting a temperature derivative

The units are °F per hour, and the sign is negative.

Final answer: **At 3 hours the temperature is falling at 4.5 °F per hour.**

Exercise 3 - Velocity and direction

$v(t) = 2t - 6$. At rest when $2t - 6 = 0$: $t = 3$.

For $0 \leq t < 3$, $v < 0$: moving left. For $t > 3$, $v > 0$: moving right.

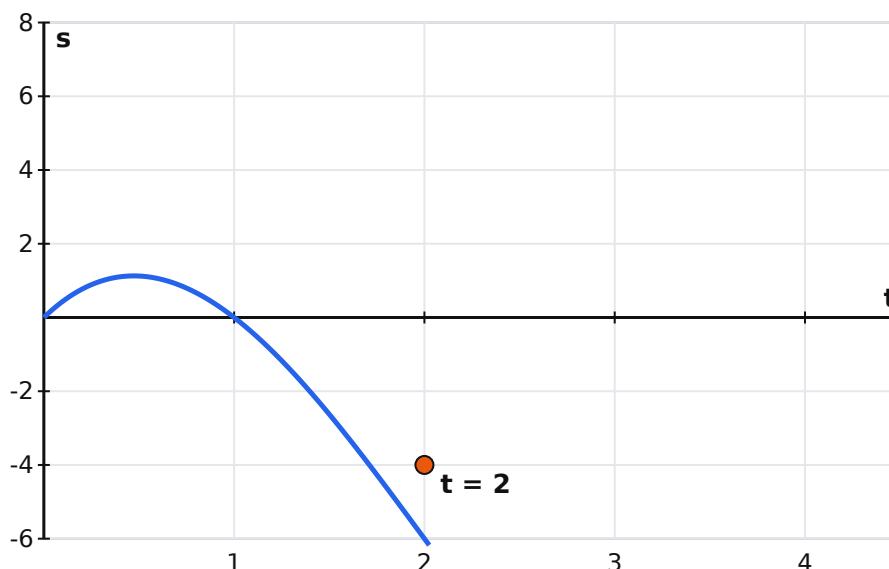
Final answer: $v(t) = 2t - 6$; at rest at $t = 3$; left on $[0, 3)$, right for $t > 3$.

Exercise 4 - Velocity and acceleration

$v(t) = 3t^2 - 12t + 5$, so $v(2) = 12 - 24 + 5 = -7$.

$$a(t) = 6t - 12, \text{ so } a(2) = 0.$$

Final answer: $v(2) = -7 \text{ ft/s}$, $a(2) = 0 \text{ ft/s}^2$.

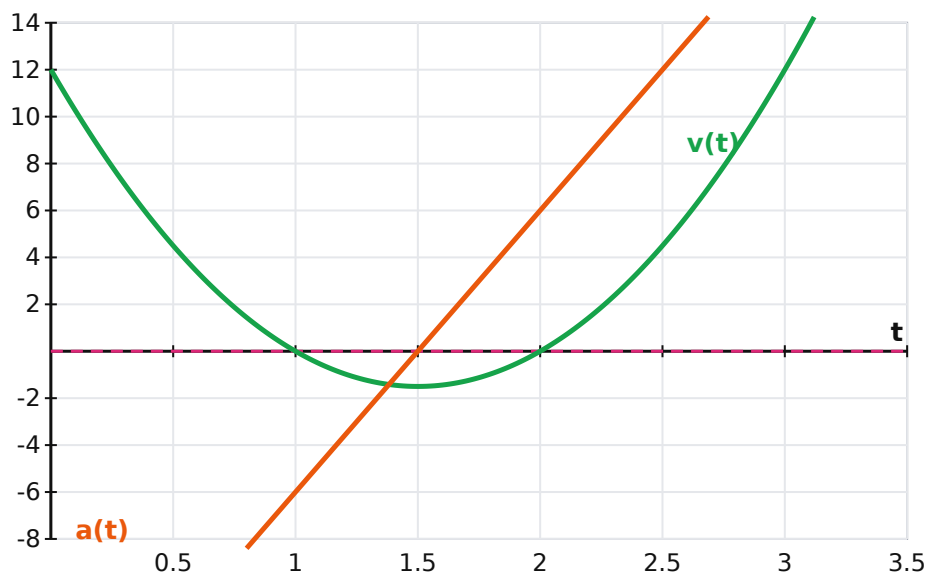


The point $t = 2$ is an inflection point: $a(2) = 0$.

Exercise 5 - Speeding up or slowing down

$$v = 6(t-1)(t-2) \text{ and } a = 12t - 18.$$

- $v(0.5) = 6(-0.5)(-1.5) = 4.5 > 0$, $a(0.5) = -12 < 0$: opposite signs, **slowing down**.
- $v(1.5) = 6(0.5)(-0.5) = -1.5 < 0$, $a(1.5) = 0$: the acceleration is zero, so the speed is momentarily constant.
- $v(3) = 12 > 0$, $a(3) = 18 > 0$: same sign, **speeding up**.



Exercise 6 - Reading a velocity graph

- a. At $t=2$ and again at $t=6.8$, where the last segment crosses zero: $-2+2.5(t-6)=0$.
- b. Velocity is negative from $t=2$ to $t=6.8$.
- c. Speed $|v|$ is 4 at $t=0$, the largest value.
- d. Acceleration is the slope of v : $\frac{0-4}{2-0} = -2 \text{ ft/s}^2$.

Exercise 7 - Sign chart for direction

$v=0$ at $t=2$ and $t=5$. Test values: $v(0)=10>0$, $v(3)=-2<0$, $v(6)=4>0$.

Final answer: right on $[0,2)$, left on $(2,5)$, right for $t>5$.

Exercise 8 - Average versus instantaneous velocity

$s(1)=3$, $s(3)=15$, so the average velocity is $\frac{15-3}{2}=6 \text{ m/s}$.

$v(t)=2t+2$, so $v(1)=4$ and $v(3)=8 \text{ m/s}$.

Final answer: 6 m/s average; 4 and 8 m/s instantaneous.

Exercise 9 - Estimating velocity from a table

$$\frac{s(3)-s(1)}{3-1} = \frac{39-5}{2} = 17.$$

Final answer: $v(2) \approx 17 \text{ ft/s}$.

Exercise 10 - Area of a circle

$$A'(r)=2\pi r, \text{ so } A'(5)=10\pi \approx 31.4.$$

Final answer: $10\pi \approx 31.4 \text{ ft}^2$ per ft: at a radius of 5 feet, each extra foot of radius adds about 31.4 square feet of area.

Exercise 11 - Volume of a cube

$$V'(s) = 3s^2, \text{ so } V'(4) = 48.$$

Final answer: 48 in^3 per inch.

Exercise 12 - Population growth

$$P'(t) = 20 - t.$$

$P'(10) = 10$: the population is growing by 10 people per year at year 10. $P'(20) = 0$: at year 20 growth stops (the population is at its maximum of 700).

Final answer: 10 people/year and 0 people/year.

Exercise 13 - Marginal revenue

$$R'(x) = 120 - 0.8x, \text{ so } R'(50) = 120 - 40 = 80.$$

Final answer: 80 dollars per skateboard.

At 50 skateboards, the 51st board adds about \$80 to the revenue.

Exercise 14 - Oscillating spring

$$v = 3 \cos t, \text{ so } v\left(\frac{\pi}{3}\right) = \frac{3}{2}. a = -3 \sin t, \text{ so } a\left(\frac{\pi}{3}\right) = -\frac{3\sqrt{3}}{2}.$$

$$\text{Final answer: } v = \frac{3}{2} \text{ cm/s, } a = -\frac{3\sqrt{3}}{2} \text{ cm/s}^2.$$

Exercise 15 - Bacteria growth

$$N'(t) = 200 \cdot 0.3e^{0.3t} = 60e^{0.3t}.$$

$$N'(5) = 60e^{1.5} \approx 268.9.$$

Final answer: $N'(5) \approx 268.9$ bacteria/hour.

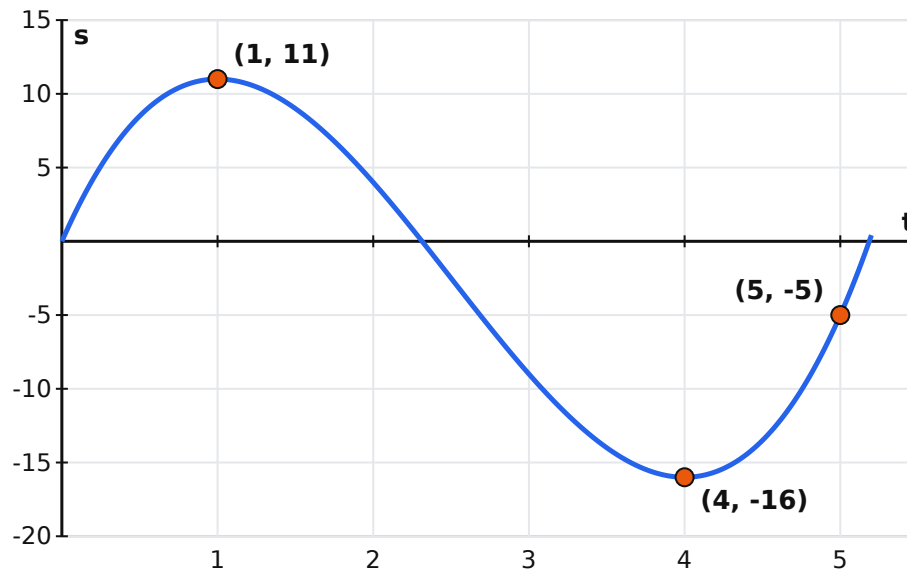
Exercise 16 - Total distance

$v = 6t^2 - 30t + 24 = 6(t-1)(t-4)$ changes sign at $t = 1$ and $t = 4$.

Positions: $s(0) = 0$, $s(1) = 11$, $s(4) = -16$, $s(5) = -5$.

Distances: $11 + 27 + 11 = 49$.

Final answer: 49 feet.



Turning points at $t = 1$ and $t = 4$.

Exercise 17 - Draining tank

Chain rule: $V'(t) = 1000\left(1 - \frac{t}{20}\right)\left(-\frac{1}{20}\right) = -50\left(1 - \frac{t}{20}\right)$.

$V'(10) = -50 \cdot 0.5 = -25$.

Final answer: -25 gallons per minute.

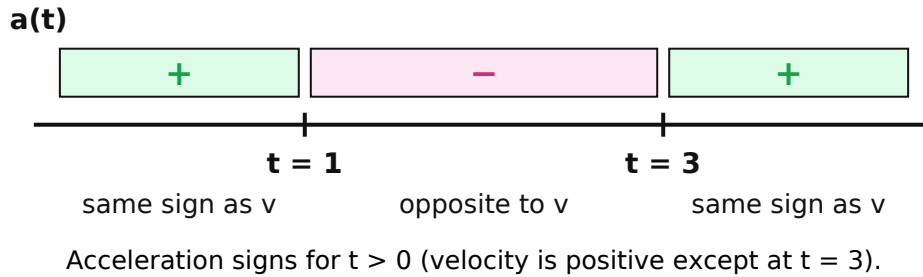
At 10 minutes the tank is losing 25 gallons per minute.

Exercise 18 - Pauses and speeding up

$v = 4t^3 - 24t^2 + 36t = 4t(t-3)^2$. At rest at $t = 0$ and $t = 3$. The factor $(t-3)^2$ never changes sign, so $v \geq 0$ for all $t \geq 0$: it never changes direction.

$a = 12t^2 - 48t + 36 = 12(t-1)(t-3)$. Speed increases where v and a have the same sign: $v > 0$ and

$a > 0$ for $0 < t < 1$ and $t > 3$.



Final answer: at rest at $t=0,3$; no direction change; speeding up on $(0,1)$ and $(3,\infty)$.

Exercise 19 - Displacement versus distance

$v = 3t^2 - 6t = 3t(t-2)$, so the cart turns around at $t=2$. Positions: $s(0)=0$, $s(2)=-4$, $s(3)=0$.

Displacement: $s(3)-s(0)=0$. Distance: $|-4-0|+|0-(-4)|=8$.

Final answer: displacement 0 m; distance 8 m.

Exercise 20 - Model rocket

$v(t) = -32t + 96$. At the top, $v=0$: $t=3$ and $s(3) = -144 + 288 + 80 = 224$ ft.

Ground: $s(t)=0$ gives $t^2-6t-5=0$, so $t=3+\sqrt{14} \approx 6.74$ s. Then $v \approx -32(6.74)+96 \approx -119.7$.

Final answer: maximum height 224 ft; $v \approx -119.7$ ft/s at impact.

Exercise 21 - Estimating a rate from a table

$$S'(2) \approx \frac{S(3)-S(1)}{3-1} = \frac{44-8}{2} = 18.$$

Final answer: $S'(2) \approx 18$.

At 2 months, sales are growing at about 18 thousand dollars per month.