

AP Calculus AB - Related Rates - Lesson

Common Core AP CALC AB 4.4 · maths-pdf.fr/en/

Related rates problems link the rates of change of quantities that are tied together by an equation. Draw and label a picture, write an equation relating the variables, differentiate with respect to time using the chain rule, and then substitute the values at the instant in question. Keep the signs and give units in the final answer.

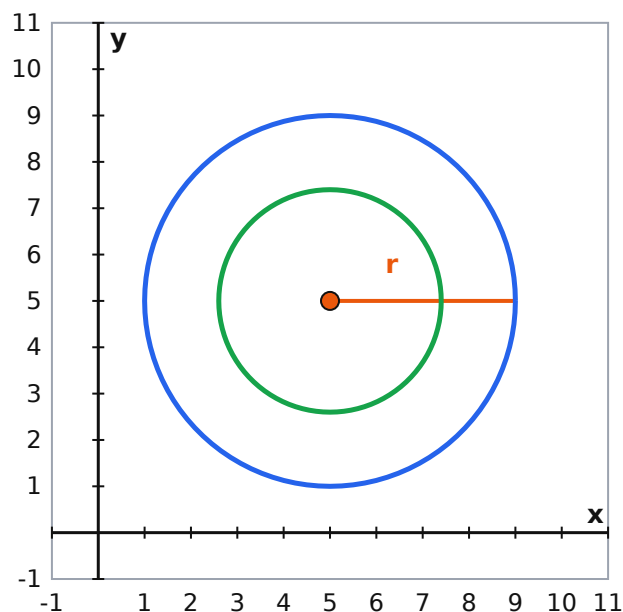
AP Calculus AB · Common Core State Standards: **AP CALC AB 4.4**

Key facts

- Differentiate the relating equation with respect to time t , using the chain rule on every variable.
- Substitute instant values only after differentiating; constants such as a ladder length stay constant.
- A quantity that is decreasing has a negative rate.
- Use similar triangles or a given ratio to reduce a formula to one variable before differentiating.

What is a related rates problem?

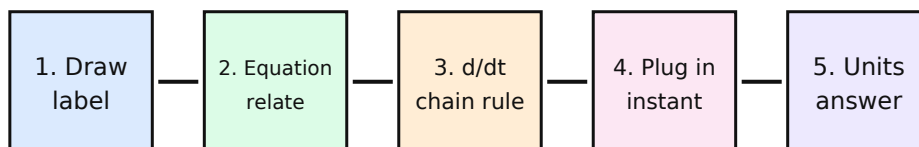
Quantities such as the radius and the area of a ripple change with time, and they are linked by an equation. If you know how fast one changes, you can find how fast the other changes. Differentiating the equation with respect to time t links the rates.



A ripple: the radius r grows with time, so the area $A = \pi r^2$ grows too.

Method

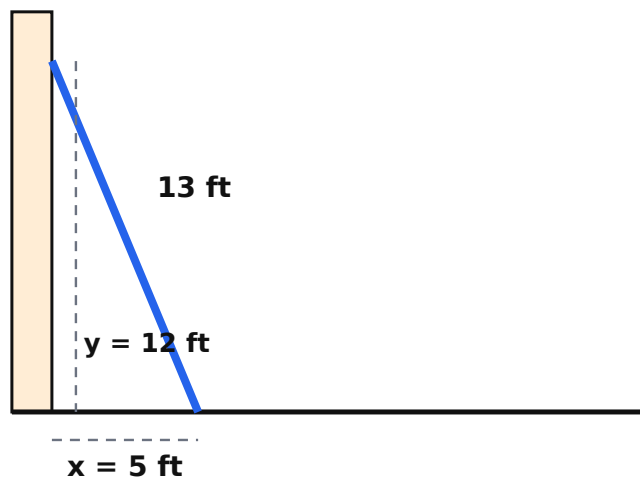
1. Draw a picture and label the changing quantities with variables.
2. Write down the given rates and the rate to find.
3. Write an equation that relates the variables.
4. Differentiate both sides with respect to t using the chain rule.
5. Substitute the values at the instant in question, solve, and give units.



Five steps for every related rates problem.

Example (ripple). A ripple has radius r feet growing at $\frac{dr}{dt} = 3$ ft/s. Since $A = \pi r^2$, $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$. At $r = 10$: $\frac{dA}{dt} = 2\pi(10)(3) = 60\pi \approx 188.5$ square feet per second.

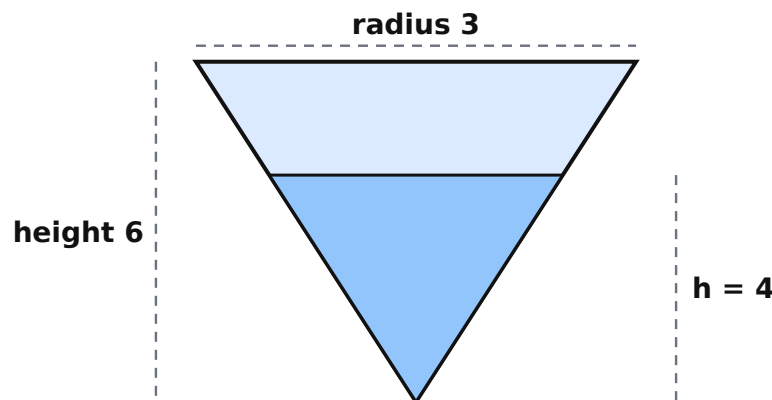
How do you set up a ladder problem?



A 13-foot ladder: $x^2 + y^2 = 169$ at every instant.

The ladder, the wall and the ground form a right triangle with $x^2 + y^2 = 13^2$. The length is constant, so differentiating gives $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$. With $x = 5$, $y = 12$ and $\frac{dx}{dt} = 2$: $10(2) + 24 \frac{dy}{dt} = 0$, so $\frac{dy}{dt} = -\frac{5}{6}$ ft/s. The negative sign means the top slides down.

What about volumes?



Cone with radius 3 in and height 6 in. At depth h the radius is $h/2$.

For a cone with radius equal to half the depth, $r = \frac{h}{2}$ gives $V = \frac{1}{3}\pi r^2 h = \frac{\pi}{12}h^3$. Reduce the equation to *one* variable before you differentiate. Then $\frac{dV}{dt} = \frac{\pi}{4}h^2 \frac{dh}{dt}$.

Example (shadow). A 6-foot person walks away from a 15-foot lamp. Let x be the distance from the lamp to the person and s the length of the shadow. Similar triangles give $\frac{15}{x+s} = \frac{6}{s}$, so $15s = 6x + 6s$ and $s = \frac{2}{3}x$. If the person walks at 4 ft/s, then $\frac{ds}{dt} = \frac{2}{3}(4) = \frac{8}{3}$ ft/s. Notice that this rate does not depend on how far away the person is.

Two warnings: do not substitute an instant value for a variable until *after* differentiating, and

keep signs: decreasing quantities have negative rates.

Questions about related rates

What is a related rates problem?

It is a problem in which several quantities change with time and are connected by an equation. Knowing one rate lets you find another by differentiating the equation with respect to time.

When do you substitute numbers in a related rates problem?

Substitute the instant values after you differentiate. If you substitute before, quantities that actually change would be treated as constants and the derivative would be wrong.

Why is the rate negative sometimes?

A negative rate means the quantity is decreasing. The top of a sliding ladder moves down, so dy/dt is negative.

What is the chain rule step in related rates?

Every variable depends on time, so differentiating y squared with respect to t gives $2y$ times dy/dt . The factor dy/dt is the chain rule term.

How do you handle two variables in a volume formula?

Use a given relationship, such as $r = h/2$ from similar triangles, to rewrite the volume in terms of one variable, then differentiate.