

AP Calculus AB - Related Rates - Answer Key

Common Core AP CALC AB 4.4 · maths-pdf.fr/en/

This answer key gives a full worked solution for every problem in the related rates worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 4.4**

Exercise 1 - Growing ripple

$$A = \pi r^2, \text{ so } \frac{dA}{dt} = 2\pi r \frac{dr}{dt}.$$

$$\text{Substitute } r = 10 \text{ and } \frac{dr}{dt} = 3: 2\pi(10)(3) = 60\pi.$$

$$\text{Final answer: } 60\pi \approx 188.5 \text{ ft}^2/\text{s}.$$

Exercise 2 - Melting ice cube

$$V = x^3, \text{ so } \frac{dV}{dt} = 3x^2 \frac{dx}{dt}. \text{ With } -6 = 3(9) \frac{dx}{dt}: \frac{dx}{dt} = -\frac{2}{9}.$$

$$S = 6x^2, \text{ so } \frac{dS}{dt} = 12x \frac{dx}{dt} = 12(3)\left(-\frac{2}{9}\right) = -8.$$

$$\text{Final answer: } \frac{dx}{dt} = -\frac{2}{9} \text{ cm/min; } \frac{dS}{dt} = -8 \text{ cm}^2/\text{min}.$$

Exercise 3 - Growing cube

$$V = x^3 \text{ gives } \frac{dV}{dt} = 3x^2 \frac{dx}{dt} = 3(9)(2) = 54.$$

$$S = 6x^2 \text{ gives } \frac{dS}{dt} = 12x \frac{dx}{dt} = 12(3)(2) = 72.$$

$$\text{Final answer: } 54 \text{ cm}^3/\text{s} \text{ and } 72 \text{ cm}^2/\text{s}.$$

Exercise 4 - Sliding ladder

$$x^2 + y^2 = 169. \text{ Differentiate: } 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0.$$

When $x=5$, $y=12$. So $10(2)+24\frac{dy}{dt}=0$ and $\frac{dy}{dt}=-\frac{5}{6}$.

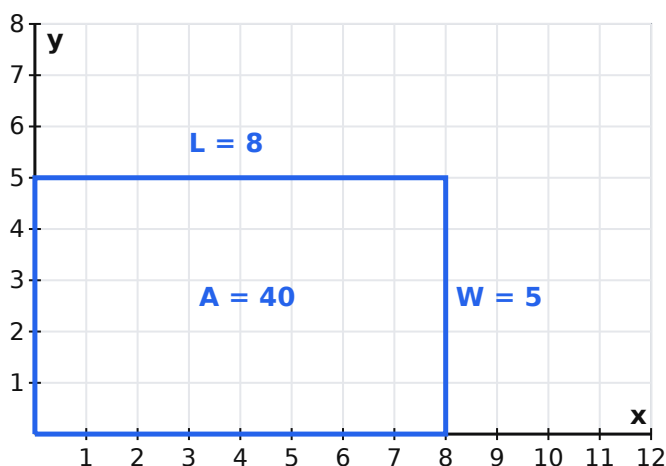
Final answer: $-\frac{5}{6}$ ft/s (the top slides down).

Exercise 5 - Rectangle

$A = LW$, so $\frac{dA}{dt} = L'W + LW' = 3(5) + 8(-2) = -1$.

$P = 2L + 2W$, so $\frac{dP}{dt} = 2(3) + 2(-2) = 2$.

Final answer: $\frac{dA}{dt} = -1$ in²/s, $\frac{dP}{dt} = 2$ in/s.



Rectangle with $L = 8$ and $W = 5$ at the instant in question.

Exercise 6 - Two cars

After 0.5 h: $y=30$ and $x=20$ miles, so $D = \sqrt{20^2 + 30^2} = \sqrt{1300} = 10\sqrt{13}$.

$D^2 = x^2 + y^2$ gives $D\frac{dD}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$, so $\frac{dD}{dt} = \frac{20(40) + 30(60)}{10\sqrt{13}} = \frac{2600}{10\sqrt{13}} = \frac{260}{\sqrt{13}} = 20\sqrt{13}$.

Final answer: $20\sqrt{13} \approx 72.1$ mph.

Exercise 7 - Cone-shaped cup

Similar triangles: $\frac{r}{h} = \frac{3}{6}$, so $r = \frac{h}{2}$. Then $V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3$.

$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$. With $h=4$: $2 = \frac{\pi}{4}(16)\frac{dh}{dt} = 4\pi\frac{dh}{dt}$.

Final answer: $\frac{dh}{dt} = \frac{1}{2\pi} \approx 0.159$ in/s.

Exercise 8 - Shadow length

Let x be her distance from the light and T the distance to the tip of the shadow. Similar triangles: $\frac{T}{15} = \frac{T-x}{6}$, so $6T = 15T - 15x$ and $T = \frac{5}{3}x$.

$\frac{dT}{dt} = \frac{5}{3}(4) = \frac{20}{3}$. The shadow length is $T - x = \frac{2}{3}x$, so it grows at $\frac{2}{3}(4) = \frac{8}{3}$.

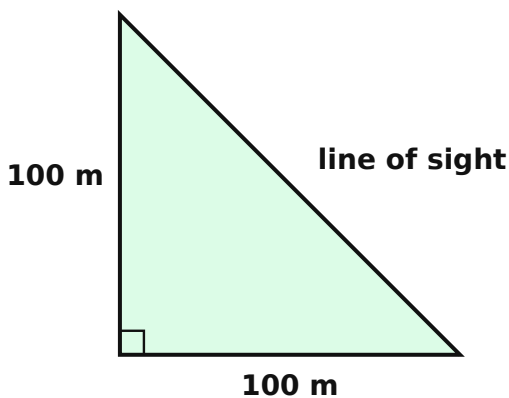
Final answer: $\frac{20}{3} \approx 6.67$ ft/s and $\frac{8}{3} \approx 2.67$ ft/s.

Exercise 9 - Angle of elevation

$\tan \theta = \frac{h}{100}$, so $\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{100} \frac{dh}{dt}$.

At $h = 100$, $\theta = \frac{\pi}{4}$ and $\sec^2 \theta = 2$. So $2 \frac{d\theta}{dt} = \frac{5}{100}$.

Final answer: $\frac{d\theta}{dt} = 0.025$ rad/s.



At $h = 100$ m the angle is 45° ($\pi/4$).

Exercise 10 - Sand pile

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h^3$. So $\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$.

$10 = \pi(25) \frac{dh}{dt}$, so $\frac{dh}{dt} = \frac{2}{5\pi}$.

Final answer: $\frac{2}{5\pi} \approx 0.127$ ft/min.

Exercise 11 - Melting sphere

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi(16)(-0.5) = -32\pi.$$

Final answer: $-32\pi \approx -100.5 \text{ cm}^3/\text{min}.$

Exercise 12 - Circle with circumference

$$C = 2\pi r, \text{ so } \frac{dr}{dt} = \frac{1}{2\pi} \frac{dC}{dt} = \frac{2}{\pi}. \text{ At } C = 20\pi, r = 10.$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(10)\frac{2}{\pi} = 40.$$

Final answer: $40 \text{ cm}^2/\text{s}.$

Exercise 13 - Gas under pressure

$$\text{At } V = 10, P = 20. \text{ Differentiate: } \frac{dP}{dt}V + P\frac{dV}{dt} = 0.$$

$$\frac{dP}{dt}(10) + 20(-3) = 0, \text{ so } \frac{dP}{dt} = 6.$$

Final answer: 6 units per second (the pressure is rising).

Exercise 14 - Draining cylinder

$$V = \pi r^2 h = 4\pi h, \text{ so } \frac{dV}{dt} = 4\pi \frac{dh}{dt}.$$

$$-12 = 4\pi \frac{dh}{dt} \text{ gives } \frac{dh}{dt} = -\frac{3}{\pi}.$$

Final answer: $-\frac{3}{\pi} \approx -0.955 \text{ ft/min}.$

Exercise 15 - Kite

$$\ell^2 = x^2 + 80^2. \text{ At } \ell = 100, x = 60.$$

$$\ell \frac{d\ell}{dt} = x \frac{dx}{dt}, \text{ so } \frac{d\ell}{dt} = \frac{60(5)}{100} = 3.$$

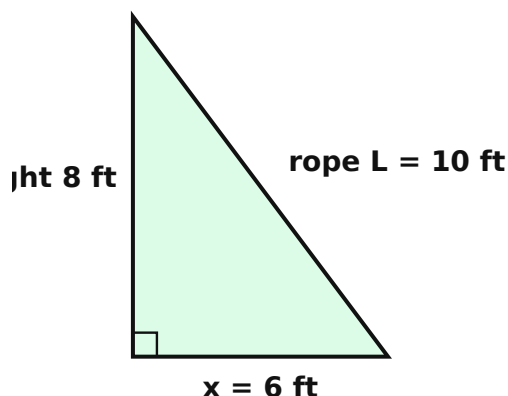
Final answer: $3 \text{ ft/s}.$

Exercise 16 - Boat and dock

$$x^2 + 64 = L^2. \text{ At } L = 10, x = 6.$$

$$x \frac{dx}{dt} = L \frac{dL}{dt}, \text{ so } 6 \frac{dx}{dt} = 10(-1.5) \text{ and } \frac{dx}{dt} = -2.5.$$

Final answer: 2.5 ft/s toward the dock.



The boat problem at the instant when 10 ft of rope is out.

Exercise 17 - Runner on a baseball diamond

$$D^2 = 90^2 + s^2. \text{ At } s = 30, D = \sqrt{9000} = 30\sqrt{10}.$$

$$D \frac{dD}{dt} = s \frac{ds}{dt}, \text{ so } \frac{dD}{dt} = \frac{30(20)}{30\sqrt{10}} = \frac{20}{\sqrt{10}} = 2\sqrt{10}.$$

Final answer: $2\sqrt{10} \approx 6.32$ ft/s.

Exercise 18 - Approaching plane

$$z^2 = x^2 + 9. \text{ At } x = 4, z = 5.$$

$$z \frac{dz}{dt} = x \frac{dx}{dt}, \text{ so } \frac{dz}{dt} = \frac{4(-480)}{5} = -384.$$

Final answer: -384 mph.

Exercise 19 - Triangle with changing angle

$$A = \frac{1}{2}(6)(8)\sin\theta = 24\sin\theta, \text{ so } \frac{dA}{dt} = 24\cos\theta \frac{d\theta}{dt}.$$

$$\text{At } \theta = \frac{\pi}{3}: 24 \cdot \frac{1}{2} \cdot 0.1 = 1.2.$$

Final answer: 1.2 cm²/s.

Exercise 20 - Oil spill

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(50)(2) = 200\pi.$$

Final answer: $200\pi \approx 628.3 \text{ ft}^2/\text{min}.$

Exercise 21 - Triangular trough

At depth h , the width of the water surface is $2h$ (similar triangles: $\frac{w}{h} = \frac{4}{2}$). The cross-section area is $\frac{1}{2}(2h)(h) = h^2$, so $V = 10h^2$.

$$\frac{dV}{dt} = 20h \frac{dh}{dt}. \text{ At } h = 1: 3 = 20 \frac{dh}{dt}.$$

Final answer: $\frac{dh}{dt} = \frac{3}{20} = 0.15 \text{ ft/min}.$

Exercise 22 - Volume and surface area

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi(36)(0.4) = 57.6\pi. \quad S = 4\pi r^2 \text{ gives } \frac{dS}{dt} = 8\pi r \frac{dr}{dt} = 8\pi(6)(0.4) = 19.2\pi.$$

$$\text{Ratio: } \frac{57.6\pi}{19.2\pi} = 3 = \frac{6}{2}.$$

Final answer: $57.6\pi \approx 181 \text{ cm}^3/\text{s}; 19.2\pi \approx 60.3 \text{ cm}^2/\text{s}.$