

AP Calculus AB - Related Rates - Test Answer Key

Common Core AP CALC AB 4.4 · maths-pdf.fr/en/

Use this answer key to score the related rates test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

AP Calculus AB · Common Core State Standards: **AP CALC AB 4.4**

Question 1 - Circle area [2 pts]

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(12)(0.5) = 12\pi.$$

Final answer: $12\pi \approx 37.7 \text{ cm}^2/\text{s}$.

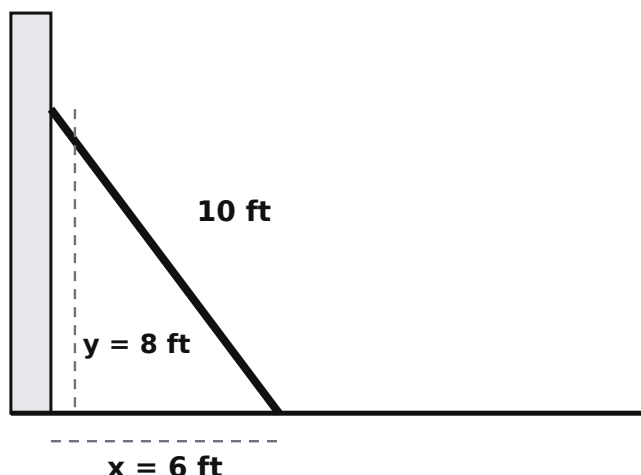
Scoring: 1 point for the differentiated equation; 1 point for the value with units.

Question 2 - Ladder [3 pts]

$$x^2 + y^2 = 100, \text{ so } x \frac{dx}{dt} + y \frac{dy}{dt} = 0. \text{ At } x = 6, y = 8.$$

$$6 \frac{dx}{dt} + 8(-1.5) = 0 \text{ gives } \frac{dx}{dt} = 2.$$

Final answer: 2 ft/s .



A 10-foot ladder with its base 6 feet from the wall.

AP Calculus AB Related Rates - Test Answer Key - Maths PDF ·

<https://maths-pdf.fr/en/ap-calculus-ab-related-rates-test-solutions-31200> · page 1/4

Scoring: 1 point for the equation and its derivative; 1 point for the values $x = 6$, $y = 8$; 1 point for the answer with units.

Question 3 - Cone volume [3 pts]

Similar triangles: $r = \frac{h}{2}$, so $V = \frac{\pi}{12}h^3$ and $\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$.

$$-6 = \frac{\pi}{4}(16)\frac{dh}{dt} = 4\pi\frac{dh}{dt}, \text{ so } \frac{dh}{dt} = -\frac{3}{2\pi}.$$

Final answer: $-\frac{3}{2\pi} \approx -0.477$ ft/min.

Scoring: 1 point for r in terms of h ; 1 point for the derivative; 1 point for the value.

Question 4 - Rectangle [2 pts]

$$\frac{dA}{dt} = \frac{dL}{dt}W + L\frac{dW}{dt} = 4(6) + 10(-1) = 14.$$

Final answer: 14 cm²/s (increasing).

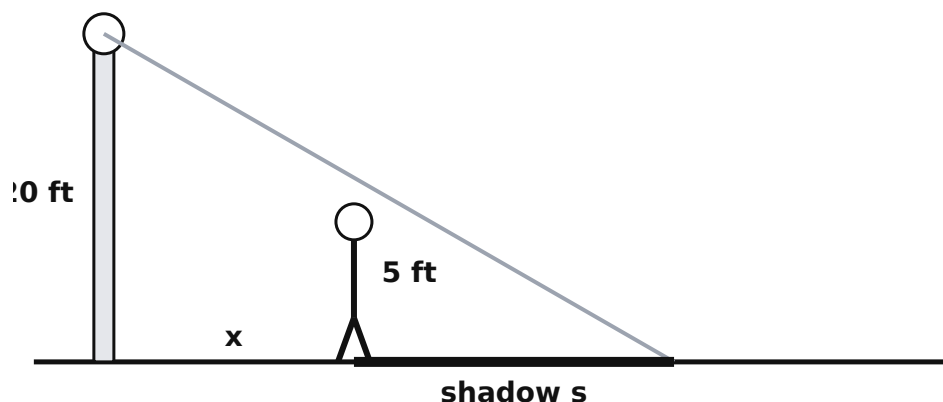
Scoring: 1 point for the product rule; 1 point for the value.

Question 5 - Shadow [3 pts]

With x the distance from the post and s the shadow length: $\frac{20}{x+s} = \frac{5}{s}$, so $20s = 5x + 5s$ and $s = \frac{x}{3}$.

$$\frac{ds}{dt} = \frac{1}{3}(6) = 2.$$

Final answer: 2 ft/s.



A 20-foot light and a 5-foot person.

Scoring: 1 point for similar triangles; 1 point for s in terms of x ; 1 point for the rate.

Question 6 - Distance between boats [2 pts]

After 1 hour: $x=12$, $y=5$, $D=13$. $D\frac{dD}{dt}=x\frac{dx}{dt}+y\frac{dy}{dt}$ gives $13\frac{dD}{dt}=12(12)+5(5)=169$.

Final answer: 13 mph.

Scoring: 1 point for the equation; 1 point for the value.

Question 7 - Angle change [2 pts]

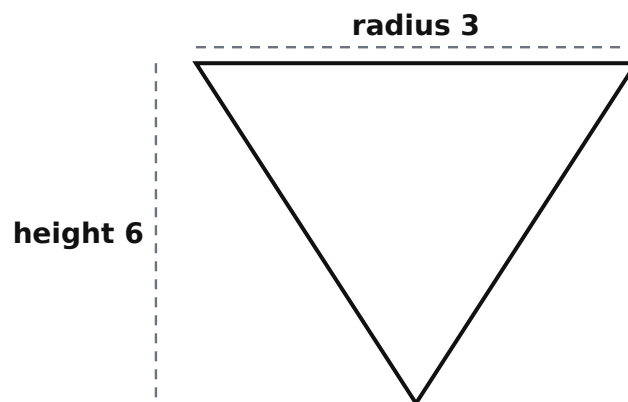
$\tan\theta = \frac{x}{50}$, so $\sec^2\theta\frac{d\theta}{dt} = \frac{1}{50}\frac{dx}{dt}$. At $\theta = \frac{\pi}{4}$: $2\frac{d\theta}{dt} = \frac{8}{50}$.

Final answer: 0.08 rad/s.

Scoring: 1 point for the differentiated equation; 1 point for the value.

Question 8 - Sphere and cylinder [3 pts]

- $\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt} = 4\pi(100)(0.2) = 80\pi \approx 251.3 \text{ cm}^3/\text{s}$.
- $\frac{dS}{dt} = 8\pi r\frac{dr}{dt} = 8\pi(10)(0.2) = 16\pi \approx 50.3 \text{ cm}^2/\text{s}$.
- The volume rate is larger (ratio $\frac{r}{2} = 5$) because volume grows with the cube of the radius while area grows with the square.



A cone-shaped paper cup.

Scoring: 1 point per part.

Total
20 points